# 3D Estimates of Analysis and Short-Range Forecast Error Variances

**Jie Feng, Zoltan Toth**

Global Systems Division, ESRL/OAR/NOAA, Boulder, CO, USA

**Malaquias Peña**

Environmental Modeling Center, NCEP/NWS/NOAA, College Park, MD, USA

**Acknowledgment to**

**Hongli Wang, Yuanfu Xie, Scott Gregory and Isidora Jankov**

7th EnKF Data Assimilation Workshop

PSU, 2016-05-25

# Outline

- Motivation
- Method and experimental setup
- Error estimation in QG model OSSE
- Preliminary results from GFS model
- Discussions and future work

# Motivation

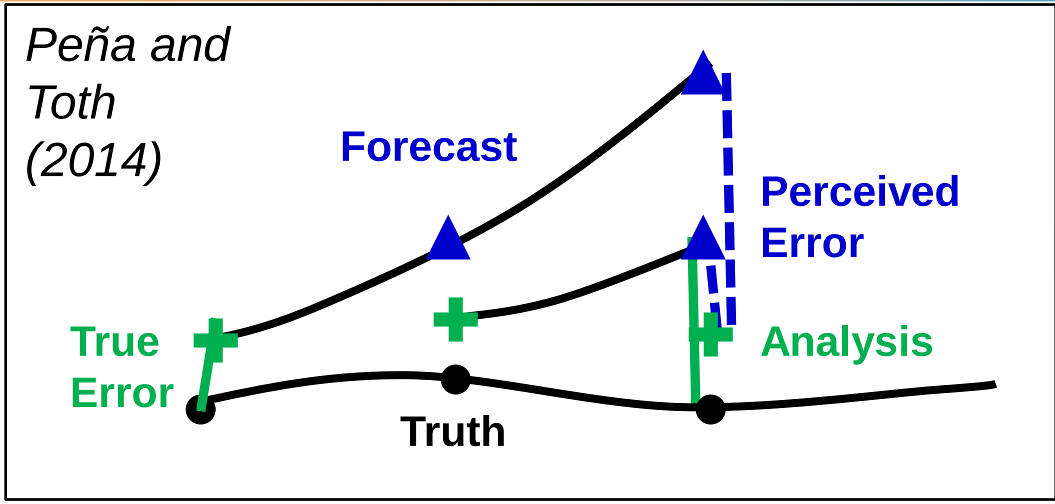
**Accurate estimates of error variances in numerical analyses and forecasts are critical:**

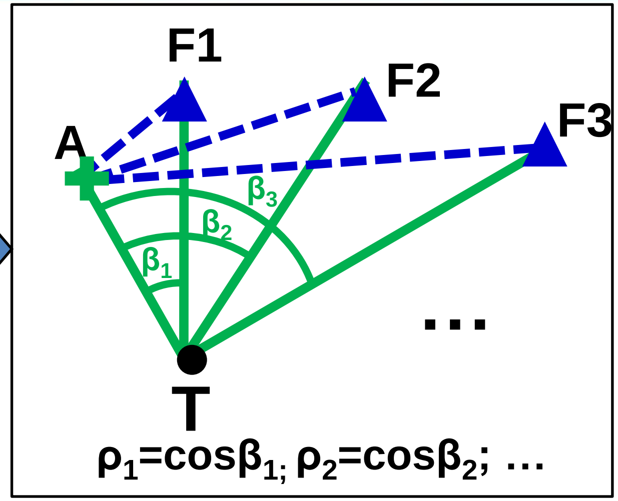
- Evaluation of forecast system
- Tuning of data assimilation (DA) system
- Proper initialization of ensemble forecasts

## Traditional methods:

- **Observations as proxy**
  - Sparse observations – no gridded information
  - Fraught with observational error (including representativeness error)
- **DA schemes themselves**
  - Computationally expensive
  - Affected by same assumptions used in DA scheme, potentially biased/inaccurate estimates
- **Short-range forecasts (forecast minus analysis)**
  - Ignore model forecast related uncertainties

# Statistical Analysis and Forecast Error (SAFE) Estimation




Forecast State

Analysis State

Analysis Error

$$d_i^2 = (F_i - A)^2 = ((F_i - T) - (A - T))^2 \equiv (x_i - x_0)^2$$

Perceived Error

True State

Forecast Error

**Measurements** $\underline{d_i^2} = \underline{x_0^2 + x_i^2 - 2\rho_i x_0 x_i}$ **Estimated quantities**

**Can we estimate unknown parameters with observed quantities?**

# Cost Function and Relevant Assumptions

$$\underbrace{d_i^2}_{\text{Measurements}} = \underbrace{x_0^2 + x_i^2 - 2\rho_i x_0 x_i}_{\text{Estimated quantities}}$$

**Cost Function** $J = \max(|d_i^2 - \hat{d}_i^2|) \cdot w_i^{-1}$

- max : $L_\infty$ norm
- Sampling standard error of the mean (SEM)
- Minimization: Limited-memory BFGS

$$SEM_i = \frac{sd_i}{\sqrt{N}} \cdot f$$

$$f = \sqrt{(1+r_1)(1-r_1)}$$

$$w_i = \frac{SEM_i}{\sum_i SEM_i}$$

*Peña and Toth (2014)*

## Connect measurements to estimates:

(1) How true error grows in time; →

(2) How true forecast errors get decorrelated from true analysis errors with increasing lead time.

$$\rho_i = \rho_1^{\,i}$$

**Exponential**

$$x_i^2 = x_0^2 e^{\alpha t_i}$$

**Logistic**

$$x_i^2 = \frac{S_\infty \cdot c}{e^{-\alpha t_i} + c}$$

$$c = x_0^2 / (S_\infty - x_0^2)$$

<u>**α : Growth Rate**</u>

<u>**$S_\infty$ :Saturation Value**</u>

# Analysis / Forecast Error Correlation

- With no DA step, analysis & forecast errors correlate at 1.0

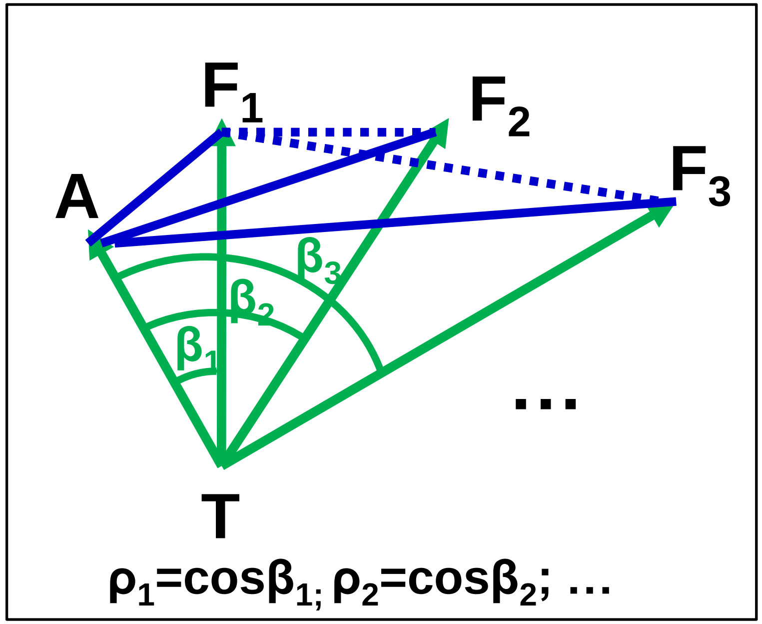
- With one DA step, errors become de-correlated, $1 > \rho_1 > 0$;

- With multiple (*i*) DA steps,

  -Assuming effectiveness of observing & DA systems stationary in time

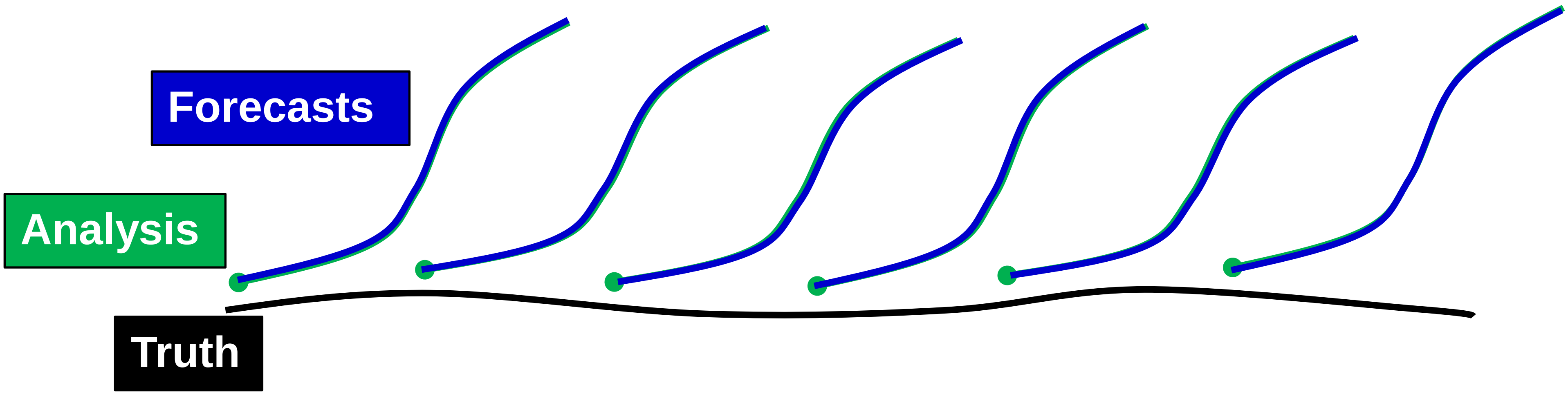
- Note same analysis system used for both Initialization & verification

$$\rho_i = \rho_1^{\ i}$$



# Experimental Setup

**Perfect model OSSE environment - Truth is known; Develop and test SAFE method that can be used in real world environment (w/o knowing truth).**




**Model: Quasi-geostrophic model (T21L3; *Marshall and Molteni, 1993*)**

**DA: Ensemble Kalman Filter (EnKF)**

- **200-member ensemble;**
- **1.69 inflation of background covariance, no localization;**

**Setup: 30-day forecast every 12hrs over 90-day period (180 cases).**

# Exponential Error Growth

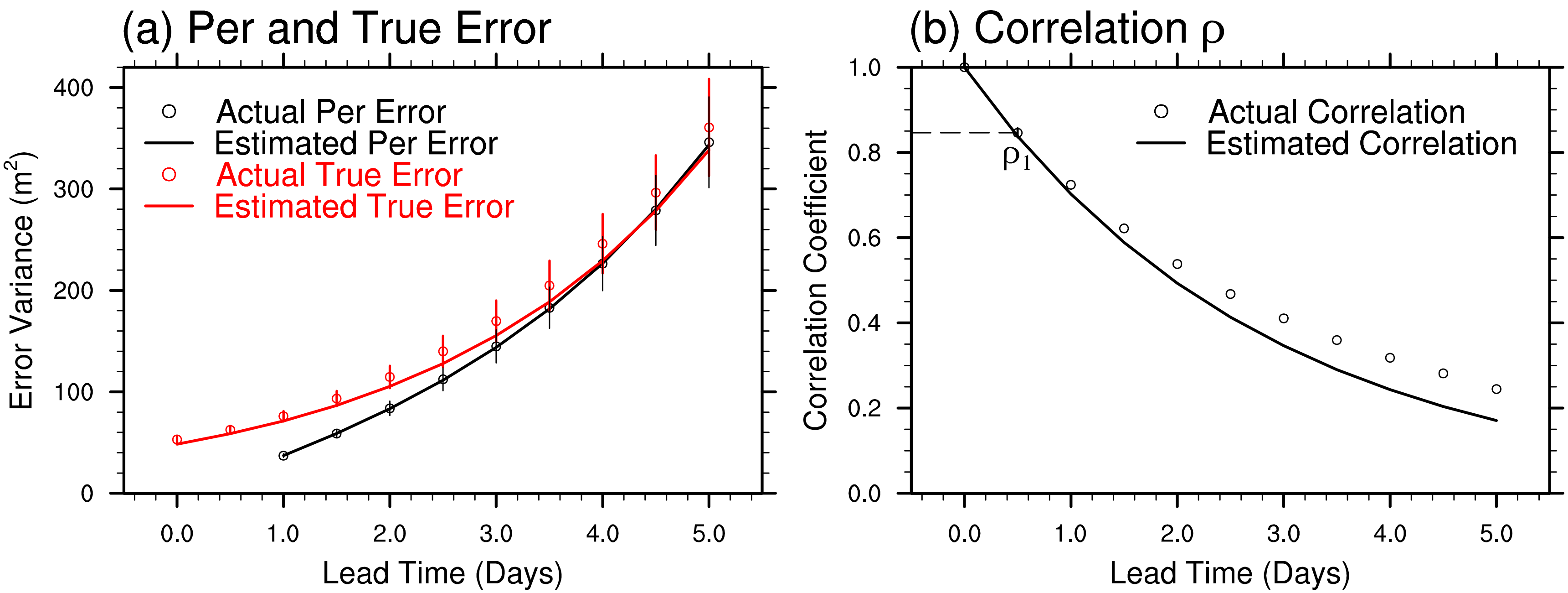
**3D spatial and temporal mean error variance of GHT500**

$$\langle(\cdot)\rangle = \frac{1}{m \cdot n}\sum_{i=1,\, j=1}^{m,n} \frac{1}{t}\sum_{k=1}^{t}(\cdot)$$

| | $x_0^2$ | $\alpha$ | $\rho_1$ |
|---|---|---|---|
| **Actl** | 53.0, | 0.38, | 0.85 |
| **Est** | 48.4, | 0.39, | 0.84 |

**$x_0$ 4% difference**

**1.96*SEM~95% confidence interval (uncertainty bar)**




- **Assumptions consistent with data**
- **Differences between measured and modeled values may because:**
  - **(1) Initial decay of analysis error not presented in SAFE;**
  - **(2) Linear exponential growth is an approximation;**
  - **(3) Sampling errors of finite samples**

# Grid-Point Error Estimation

## Key Points

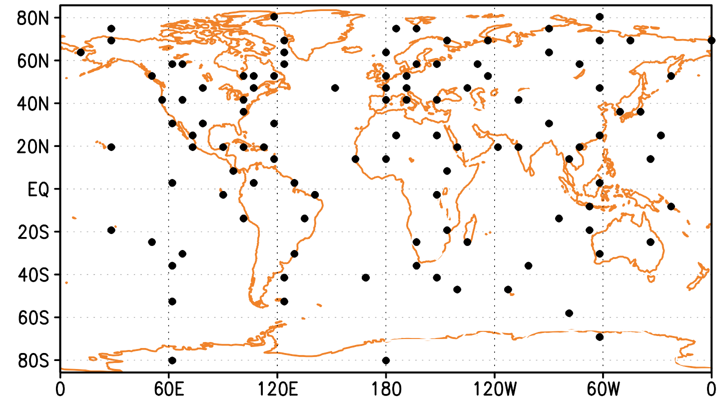
(1) Much smaller sample size, noisier input data, more difficult estimation;
(2) $\rho_1$ varies in space with the observing network and the DA scheme, present large-scale characteristics.

## Practical approach

Step1. Estimate $\rho_1$ using spatially smoothed data;
Step2. Estimate other parameters with $\rho_1$ specified from spatially smoothed estimates.

| | | GB | NH | SH | TRO |
|---|---|---|---|---|---|
| $X_0^2$ | Actl | 42.23 | 32.12 | 72.82 | 25.32 |
| | Etm | 40.26 | 33.64 | 69.43 | 20.21 |
| $\alpha$ | Actl | 0.405 | 0.574 | 0.297 | 0.300 |
| | Etm | 0.405 | 0.567 | 0.281 | 0.326 |
| $\rho_1$ | Actl | 0.840 | 0.789 | 0.859 | 0.860 |
| | Etm | 0.841 | 0.787 | 0.853 | 0.840 |

$\rho_1$ varies only moderately: 0.78-0.86




Estimated spatial mean $\rho_1$ of GHT500 over GB, NH, SH and TRO are all within 95% confidence interval (1.96*SEM of $\rho_1$).

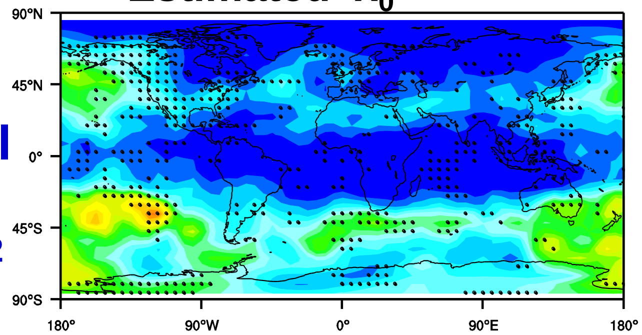
# Practical Estimation

**Black dots:** estimates are out of 95% level defined by 1.96*SEM of $x_0^2$ **23%**

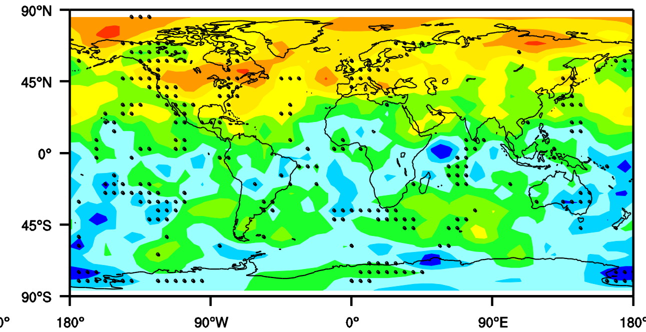
**Spatial corr with the truth:** $x_0^2$ 0.81

**Global mean of grid-point value:** $x_0^2$ 40.0 / 42.2

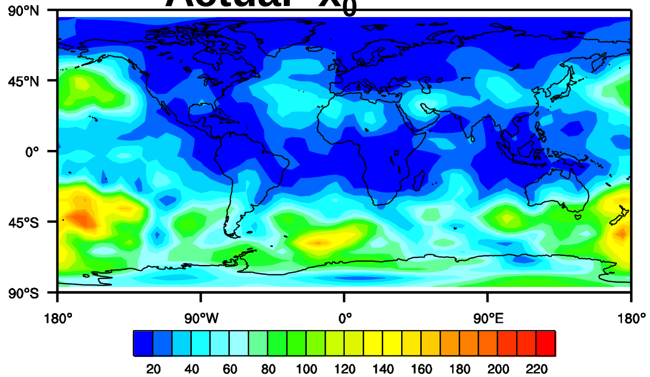
### Estimated $x_0^2$



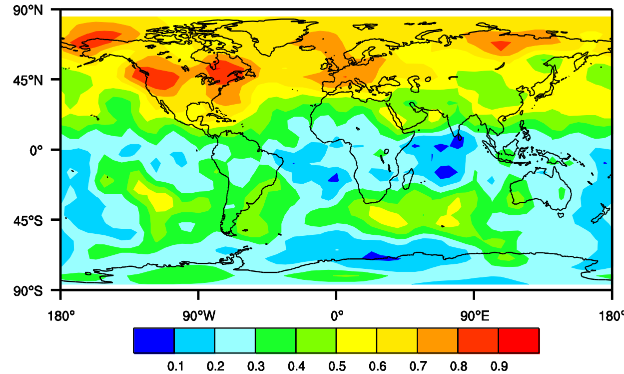

### Estimated α



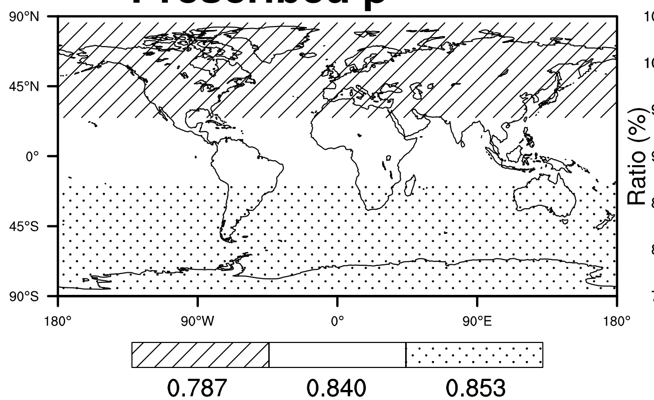

### Actual $x_0^2$



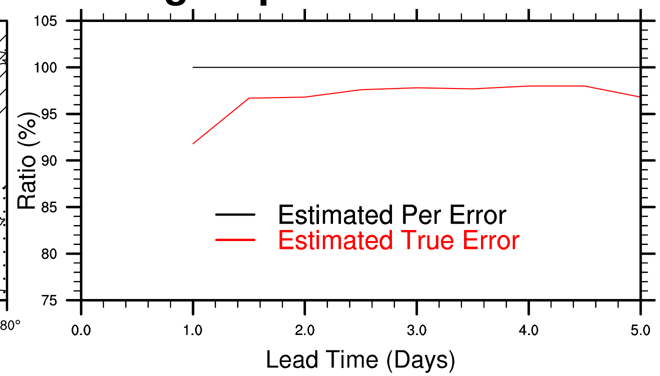

### Actual α




### Prescribed ρ




### Ratio of grid-point within 95% level




**Black dots:** estimates are out of 95% level defined by 1.96*SEM of α **16%**

**Spatial corr with the truth:** α 0.85

**Global mean of grid-point value:** α 0.37 / 0.38

# Error estimation in GFS operational forecasts

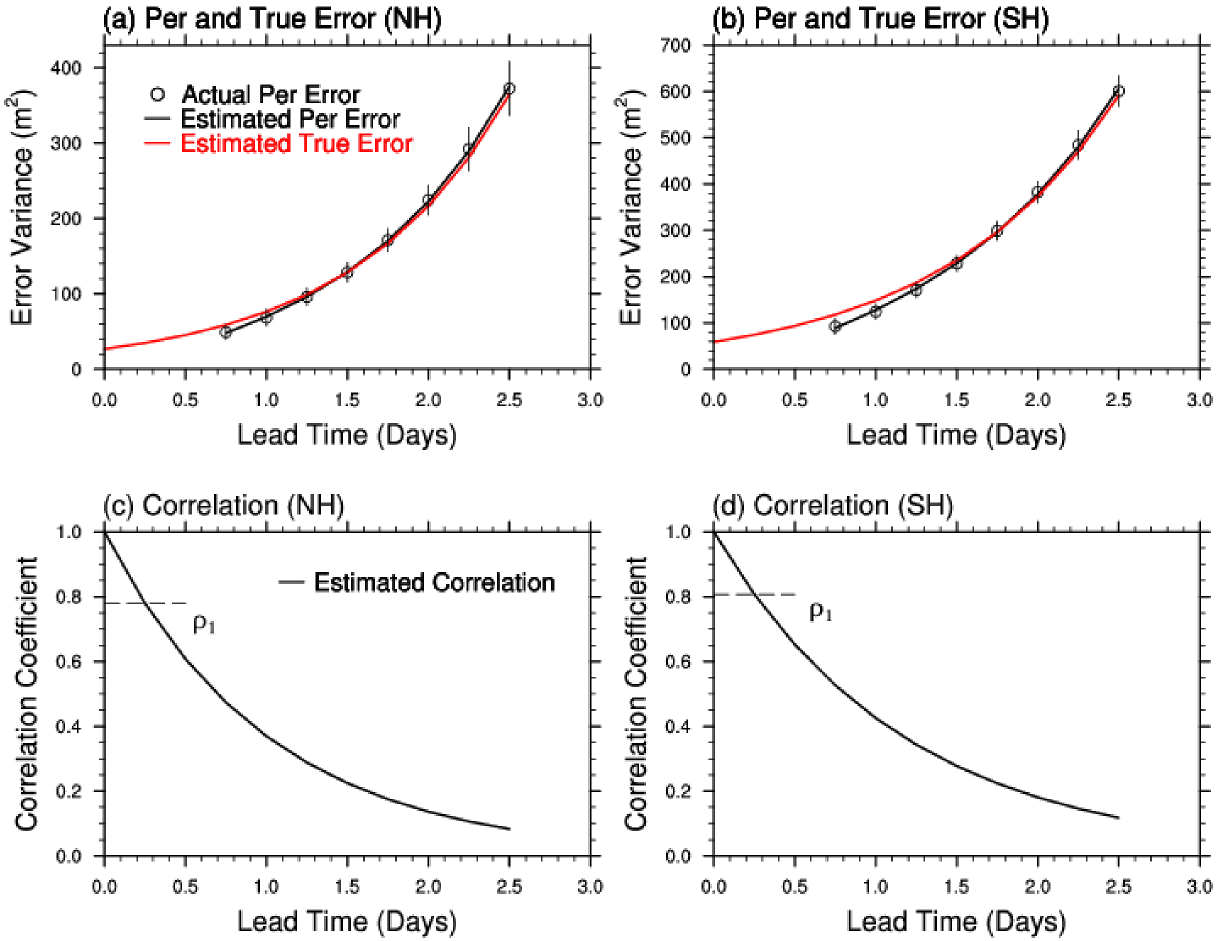
**Period: 1Sep-30Nov, 2015; Variable: GHT500; Spatial Resolution: 1°X1°**

**NH (30°-90°)**
**True Error**
**Perceived Error**

**SH (30°-90°)**
**True Error**
**Perceived Error**

**NH (30°-90°)**
**Correlation ρ**

**SH (30°-90°)**
**Correlation ρ**




| | NH | SH |
|---|---|---|
| $x_0^2$ | 26.9 | 59.0 |
| α | 1.04 | 0.92 |
| ρ | 0.78 | 0.81 |

- **Analysis error variance NH<SH;**
- **Error growth rate NH>SH, NH stronger baroclinic instability**
- **ρ SH>NH, sparser observations**

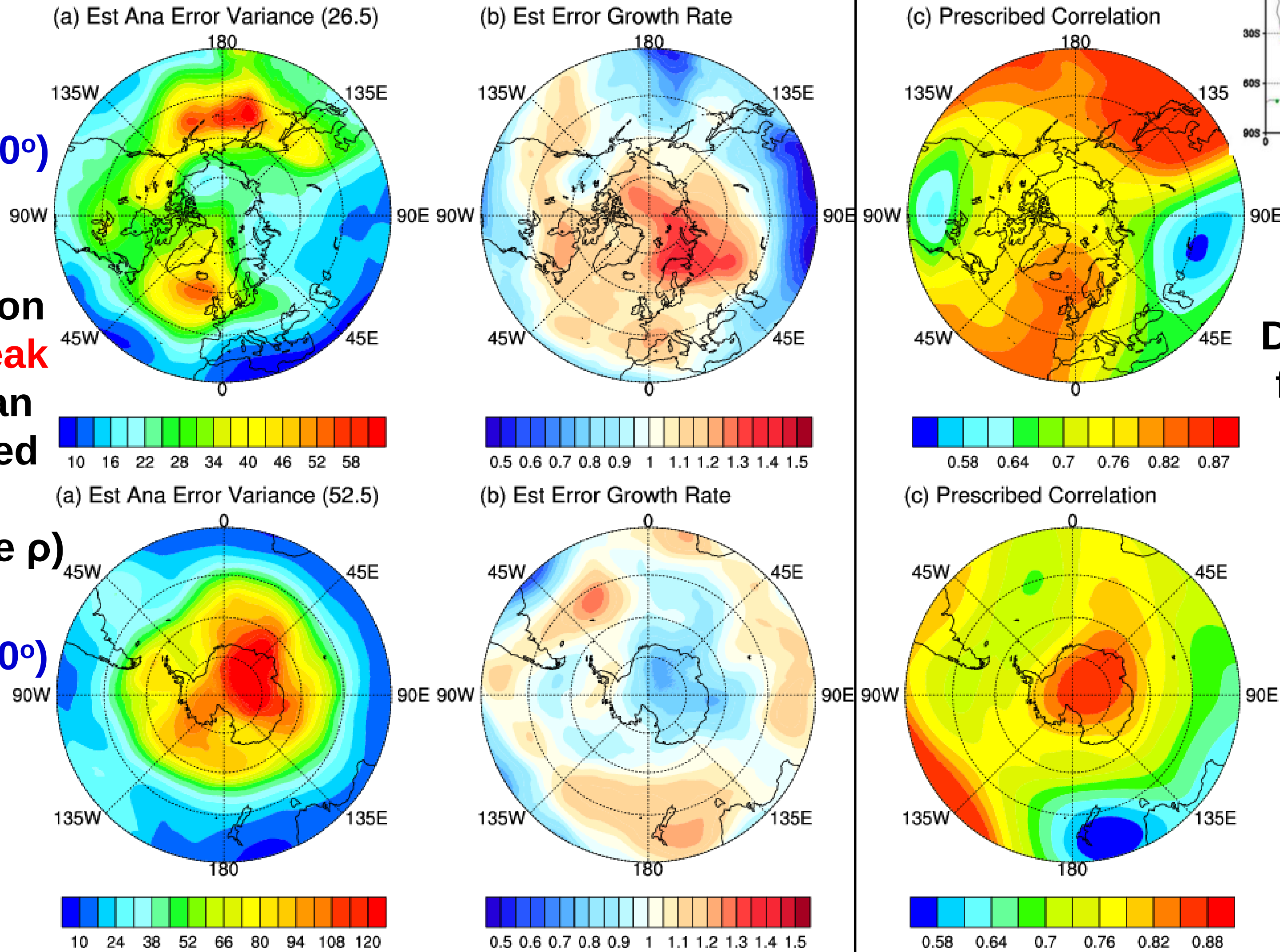
# Grid-point Error estimation of 500hPa GH

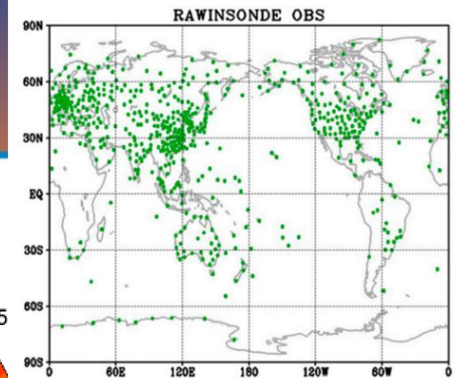
**Ana Error Variance** | **Error Growth Rate** | **Prescribed ρ**

**NH (30°-90°)**

**Estimation from** **weak** **Gaussian smoothed data (prescribe ρ)**

**SH (30°-90°)**




*Kleist and Ide (2015)*

**Direct estimated ρ from very strong Gaussian smoothed data**

- **$x_0^2$ and ρ are closely related to the observational network.**
  **NH: evident land & ocean difference**
  **SH: basically zonal distribution**
- **large α is related to polar & subtropical jet stream.**

**Estimated perceived errors at each grid point for all 2.5dy lead time are within 95% confidence interval**

# Ongoing and Future Work

**Assessment of statistical deviation from unknown truth may be possible with some accuracy. The SAFE is cheap and independent of each DA scheme.**

- **Describe initial decay of random analysis error variance in error growth model to improve accuracy of estimates;**
- **Spatial mean and 3D grid-point estimation of GFS total energy, wind, temperature, etc. other variables;**
- **Application areas:**

**(1) Specify first guess error variance in any DA scheme.**
**(2) Set initial ensemble variance in any ensemble generation scheme.**

# THANKS

# Comparison with EnKF & NMC error estimates

**EnKF (ensemble spread) — Estimates of analysis and forecast error variance**
**NMC —— Estimates of background forecast error variance**

| | | EnKF | NMC | SAFE |
|---|---|---|---|---|
| Actl Analysis Error Variance($m^2$): 42.2 | Spatial Corr | 0.92 | NA | 0.90 |
| | Error Variance($m^2$)/ Deviation of Est | Before inflation: 19.0/55% After inflation: 32.1/24% | NA | 39.8/6% |
| Actl Background Error Variance12h ($m^2$): 50.3 | Spatial Corr | 0.90 | 48hr–24hr : 0.63 24hr–12hr : 0.78 | 0.87 |
| | Error Variance($m^2$)/ Deviation of Est | Before inflation: 22.8/55% After inflation: 38.6/23% | 48hr–24hr : 48.9/ 3% 24hr–12hr : 18.9/ 62% | 47.5/6% |

- **EnKF: spatial distribution (good), magnitude (severely underestimated);**
  - **Correlation may be lower when used with other DA schemes (e.g., hybrid GSI)**
- **NMC: spatial distribution (bad), magnitude (good, tuned in operational forecast systems);**
- **Both magnitude and spatial distribution reasonably estimated by SAFE**
  - **At very low CPU cost compared to EnKF in operational setting**
  - **Estimates independent of DA scheme used**