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Multiscale Methods for Filtering and Prediction of Complex Turbulent Systems

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Lecture Available at Majda's Website:
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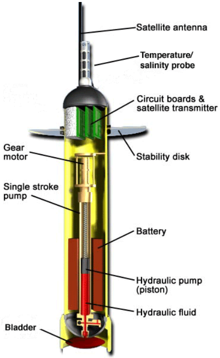
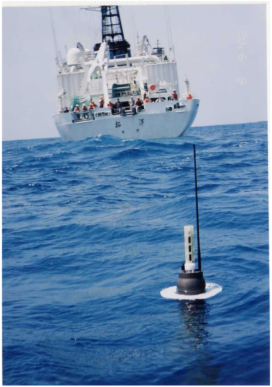
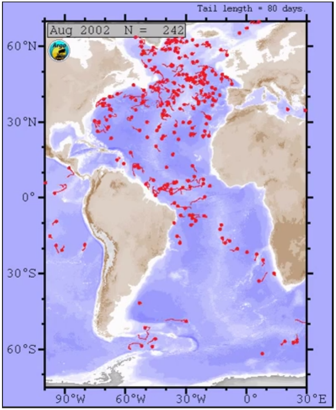
Inverse Problems and Data Assimilation

Lagrangian Tracers: Oceanography







C. Jones, A. Apte, A. Stuart, ...

Inverse Problem: Noisy Lagrangian Tracers in Filtering Geophysical Flows

First rigorous math theory

(Nan Chen, Majda, Xin Tong, *Nonlinearity* 2014, *JNLS* 2015)

Observing L noisy trajectories $X_j(t)$,

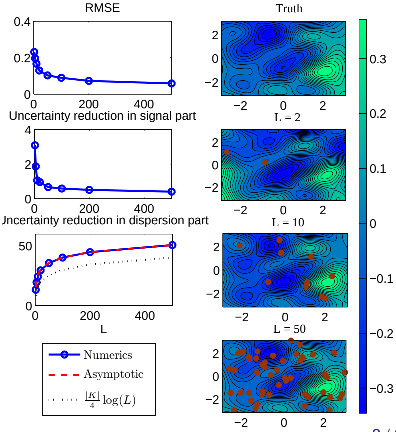
$$\frac{dX_j}{dt} = v(X_j(t), t) + \sigma_j \dot{W}_j.$$

Recover or estimate the velocity $\vec{v}$.

- Inherent nonlinearity in measurement.
- Build exact closed analytic formulas for the optimal filter for the velocity field.
- Prove a mean field limit at long times.

1. Recovering random incompressible flows

- Show an exponential increase in the number of tracers for reducing the uncertainty by a fixed amount – **a practical information barrier**.

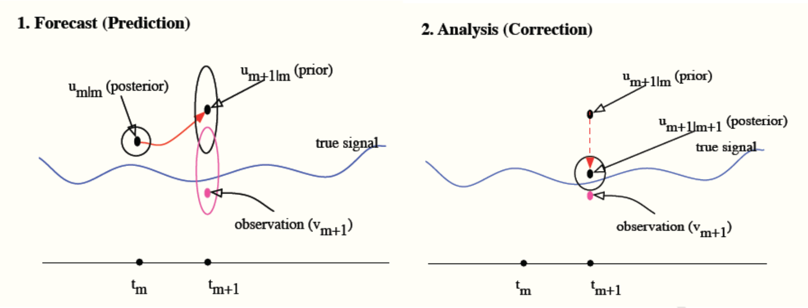


2. Noisy Lagrangian tracers for filtering random rotating compressible flows

(Nan Chen, Majda, Xin Tong, *JNLS* 2015)

- Rotating shallow water models with multiscale features:
  - Slow modes – random incompressible geostrophically balanced (GB) flows.
  - Fast modes – random rotating compressible gravity waves.
- Highly nonlinear observations mixing GB and gravity modes.
- Proposing different filters.
  - Full filter – full forecast model & tracer observations.
  - Ideal reference GB filter – GB forecast model & GB observations.
  - Reduced filter – GB forecast model & mixed observations – a practical inexpensive imperfect filter.
- Rigorous math theory: Comparable high skill in recovering GB modes for all the filters in the geophysical scenario with small Rossby number.

Filtering the Turbulent Signals



Filtering is a two-step process involving statistical prediction of the state variables through a forward operator followed by an analysis step at the next observation time which corrects this prediction on the basis of the statistical input of noisy observations of the system.

Practical Issue

- Turbulent dynamical system.
- Huge phase space, $N = O(10^6, 10^8, etc)$.
- Nonlinearity, small ensemble size $M = O(50, 100)$.

Applied algorithm

- Finite ensemble Kalman filter, (Evensen, 1995; C. Bishop, J. Anderson 2001; Kalnay, 2013). See M-H book.

Applied math

- Stuart, Reich,...

Central issues

- Why does EnKF often work well to estimate the mean with $M \leq N$?

Surprising pathology

- Catastrophic filter divergence. For filtering forced dissipative system with absorbing ball property such as L-96 model, EnKF can explode to machine infinity in finite time! (Harlim and Majda 2008; Gottwald and Majda, *NPG* 2013)

Well posedness of EnKF

- Kelly, Law, Stuart, *Nonlinearity* 2014.

Rigorous nonlinear stability for finite ensemble Kalman filter (EnKF)

(Xin Tong, Majda, Kelly, *Nonlinearity* 2015)

Filter divergence – a potential flaw for EnKF:

- Catastrophic filter divergence: the ensemble members diverging to infinity,
- Lack of stability: the ensemble members being trapped in locations far from the true process.

Finding practical conditions and modifications to rule out filter divergence with rigorous analysis:

- Ruling out catastrophic filter divergence by establishing an energy principle for the filter ensemble.
- Looking for energy principles inherited by the Kalman filtering scheme.
- Looking for modification schemes of EnKF that ensures an energy principle and preserving the original EnKF performance (Xin Tong, majda, Kelly, *Comm. Math. Sci.*, 2015).
- Verifying the nonlinear stability of EnKF through geometric ergodicity.

Rigorous example of catastrophic divergence:

- For filtering a nonlinear map with absorbing ball property (Kelly, Majda, Xin Tong, *PNAS* 2015).

Outstanding problem: Why and when is there accuracy in mean for $M \leq N$?

Need Statistically Accurate Inexpensive Forecast Models to Beat the Curse of Ensemble Size for Prediction, State Estimation and UQ

The MMT equation

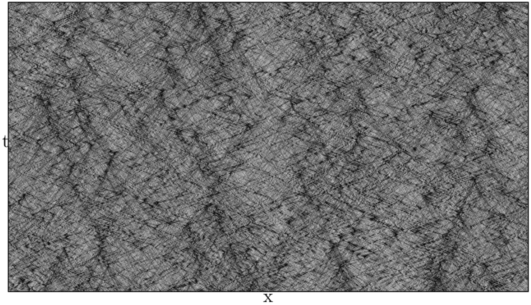
The MMT equation (Majda, McLaughlin and Tabak, 1997; Cai and M.M.T., *Phys. D* 2001)

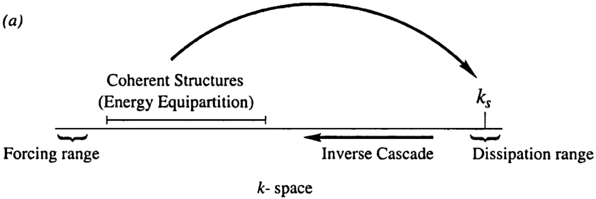
$$iu_t = |\partial_x|^{\frac{1}{2}} u + \lambda |u|^2 u - iAu + F.$$

Here we consider the case with the focusing nonlinearity, $\lambda = -1$, which induces spatially coherent 'solitonic' excitations at random spatial locations.

- The instability of collapsing solitons radiate energy to large scales producing direct and inverse turbulent cascades.
- In geophysical applications energy oftern flows from small scales to large scales (inverse cascade) creating a challenge for reduced modelling.
- Fractional dispersion are crucial with completely different behavior from NLS equation!

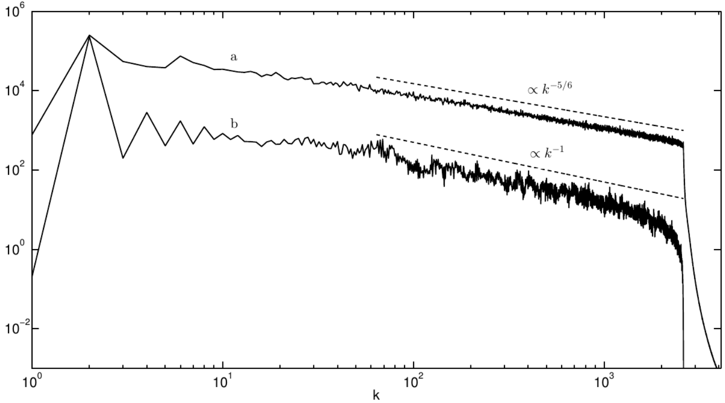
Visualization of $|\psi(x, t)|$ from simulation with $F_0 = 0.0163$; darker colors indicate higher amplitudes. Here the number of Fourier modes are $64^2 \approx 4000$.





From Cai etal, *Physica D* 2001.

High-resolution reference simulations

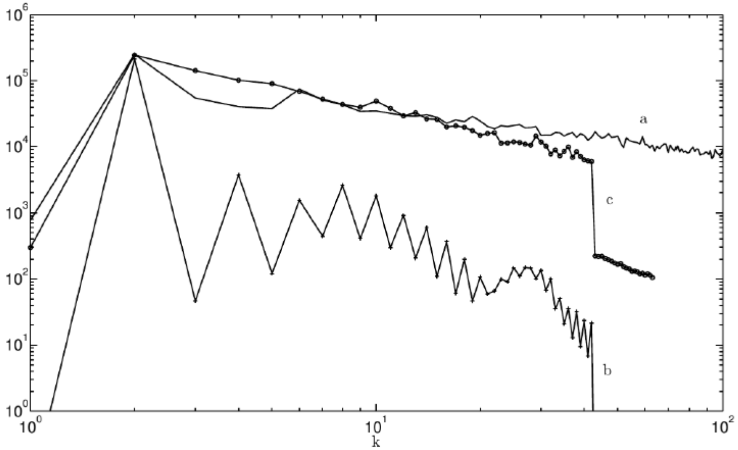


Simulation (a) uses $F_0 = 0.0163$; (b) uses $F_0 = 0.01625$. Both simulations are damped only for $2600 < |k| < 4096$ and $|k| = 1$.

Uses of MMT model:

1. Novel low order modelling: stochastic superparameterization (Majda and Grooms, *JCP* 2013; Grooms and Majda, *Comm. Math. Sci.* 2014).
2. Novel data assimilation (Branicki and Majda, *JCP* 2012; Grooms, Lee and Majda, *JCP* 2014)
3. Extreme event prediction (Cousins and Sapsis, *Phys. D.* 2014)

Stochastic Superparameterization in MMT



Spectra from simulations with $1/64$ as many points as the reference simulation (a), with no eddy terms (b) and with eddy terms (c).

Stochastic Superparameterization

1. A general framework for stochastic subgridscale modelling with no scale separation and no small-scale equilibration based on the Gaussian closure approximation and the point approximation.
2. Success in a difficult test problem with no scale separation ($k^{-5/6}$ spectra), coherent structures, dispersive waves, and an inverse cascade from unresolved scales into the large scales.
3. Overcome *curse of ensemble size* with judicious model error.

– See research expository article Majda and Grooms, *JCP* 2013; Grooms and Majda, PNAS, *JCP* 2013 for geophysical turbulence.

– See Khouider, Biello and Majda, *Comm. Math. Sci.* 2010; Deng, Khouider and Majda, *JAS* 2015 for stochastic multi-cloud model.

Multiscale Data Assimilation in Complex Turbulent System

Superparameterization (SP) and Multiscale Data Assimilation.

- Tremendously large dimension of turbulent signals requires cheap and robust coarse models for real prediction skills.
- SP is a cheap and robust under-resolved forecast model; approximates the large scale dynamics and provides small-scale statistics to estimate both the resolved and unresolved components of the true signal.
- Multiscale data assimilation framework – provides the estimate for the large-scale dynamics using SP as a coarse forecast model and partial observations of the true signal.
- Multiscale data assimilation shows robust filtering performance with a huge computational savings; better performance than other ad hoc approaches in the conventional (single-scale) data assimilation such as covariance inflation.

(Harlim & Majda, *SIAM. J. MMS*, 2013; Grooms, Lee & Majda, *JCP* 2014, *MWR* 2015; Lee & Majda, *SIAM. J. MMS*, 2015)

Blended particle filters for large dimensional chaotic dynamical systems.

Goal: Developing statistically accurate particle filters to capture non-Gaussian features in large dimensional chaotic dynamical systems.

- Space decomposition $\mathbf{u} = (\mathbf{u}_1, \mathbf{u}_2),\ u_j \in \mathbb{R}^{N_j},\ N_1 + N_2 = N,\ N_1 \ll N.$
- Blended filters:
  - Particle filter – non-Gaussian statistics of $\mathbf{u}_1$.
  - Kalman filter – conditional Gaussian statistics $\mathbf{u}_2$ given $\mathbf{u}_1$.
- Attractive feature – adaptively change of the subspaces as time evolves in response to the uncertainty without a separation of time scales using nonlinear statistical forecast models.

Nonlinear statistical forecast models:

- QG-DO – quasilinear Gaussian dynamical orthogonality method.
- MQG-DO – more sophisticated modified QG-DO method.

(Majda, Di Qi & Sapsis, *PNAS* 2014; Di Qi & Majda, *Phys D.* 2015; Sapsis & Majda, *Phys D.* 2012; *PNAS* 2013)

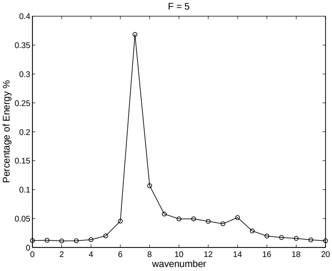
Lorenz 96 system

The Lorenz 96 system is a discrete periodic system described by the equations

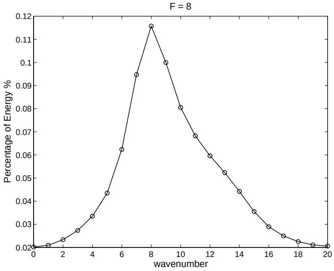
$$\frac{du_j}{dt} = (u_{j+1} - u_{j-2})u_{j-1} - u_j + F, \qquad j = 0, \ldots, J-1,$$

with $j = 40$ the number of grids and F_i the deterministic forcing. See Majda & Harlim book (2012). The quadratic part conserves energy. We will study the case of weakly chaotic turbulence ($F = 5$), strongly chaotic turbulence ($F = 8$).

5 dim subspace of particles is used.



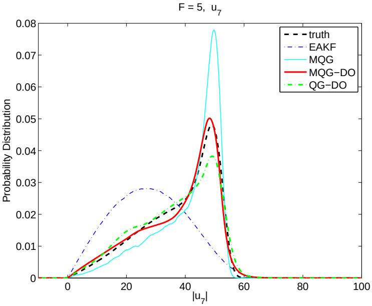
(a) $F = 5$

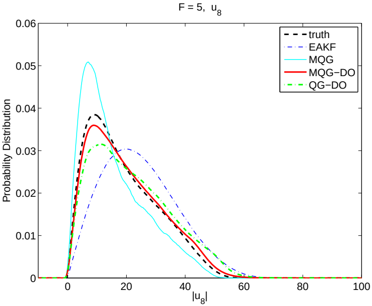


(b) $F = 8$

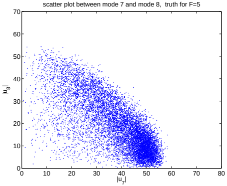
Figure: Energy Spectra

Capturing non-Gaussian statistics $F = 5, r_0 = 2, \Delta t = 1, p = 4$.

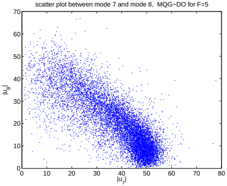




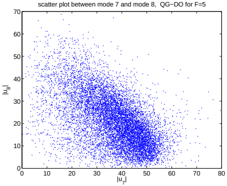
Regime scatter plot: mode u_7, u_8



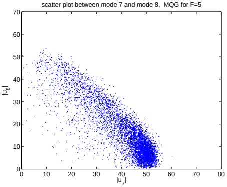
(c) truth



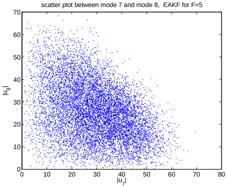
(d) MQG-DO



(e) QG-DO



(f) MQG



(g) EAKF

Stochastic Parameterized (Nonlinear) Extended Kalman Filter (SPEKF) and Dynamic Stochastic Superresolution (DSS)

- Cheap stochastic forecast models with judicious model error which are statistically exactly solvable and learn stochastic parameters “on the fly” from data
- DSS exploits SPEKF together with aliasing to achieve superresolution for subgrid scale filtering

References:

- Majda and Harlim, Filtering Complex Turbulent Systems (*Cambridge press* 2012)
- Keating, Majda and Smith, Ocean turbulence (*MWR* 2012)
- Branicki and Majda, Intermittency, black swans, wave turbulence (*JCP* 2012)
- Nan Chen, Giannakis, Majda and Herbei, MCMC algorithm for intermittency (*SIAM/ASA JUQ* 2014)
- Branicki and Majda, Turbulent Navier-Stokes (*JCP* 2016)

Thank you