A theory for gravity wave generation from jets, fronts and convection

- Derivation of a three-part formula and their validation with moist baroclinic life cycle simulations -

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Motivation:

- GWs: dynamic link between lower and middle atmosphere
- GCMs need them parameterized, empirical formulations still contain *tuning constants*
- Unified theoretical approach is needed, in particular for nonorographic GWs
- Mesoscale model simulations of moist baroclinic life cycles are used for validation

IAP

Leibniz Association

Introduction

Inertia-gravity wave (IGW) parameterizations:

• orographic: well-established (Lott & Miller, 1997)

• fronts and convection: empirical formulae available (Charron & Manzini, 2002; Chun & Baik, 1998)

• jet streaks: suggested empirical formula (Zülicke & Peters, 2006 and 2008)

$$\frac{du_h}{dt} = -fu_c = f\frac{uv_a - vu_a}{u_h} \qquad A_{*,\mathrm{cross}} = \rho_{\mathrm{mean}} \frac{([u_c] > 0)^2}{\omega_{\mathrm{mean}}}$$

Relation between cross-stream ageostrophic wind (left) and wave action (right) from Zülicke & Peters (2008), built on ideas of Uccelini & Koch (1987), Koch & Dorian (1988)…

➔ Develop a unified theoretical approach for non-orographic IGW generation.

Theory

balanced flow

imbalances

V_g
large-scale flow

V_a
„slaved“ flow

V_i
radiated waves

QGT
Quasi-Geostrophic Theory

$$\frac{d_g V_g}{dt} \propto f V_a$$

FTF
Faster-than-f condition

$$\omega_a = 2\pi \frac{V_a}{L_h} > f$$ intrinsic time scales

$$Ro_a = \frac{V_a}{f L_h} > \frac{1}{2\pi}$$ Lagrangian ageo. Rossby number

LA
Linear Approximation

$$Q \propto \frac{\partial \theta_0}{\partial z} W_a$$

Hypothesis: All too-fast “slaved” flow is radiated away with

$$E_i = \left(V_a > V_{th} = \frac{f L_h}{2\pi} \right)^2$$

Jet streaks

Horizontal motion

$$\frac{d_g u_g}{dt} = f v_a$$

$$\frac{d_g v_g}{dt} = -f u_a$$

→ jet streak exit (Langrangean tangential deceleration)

$$\frac{d_g V_g}{dt} = -D^{jet} = -f V_a^{jet}$$

$$V_g = \left(u_g^{\,2} + v_g^{\,2}\right)^{1/2}$$

"slaved flow"

$$V_a^{jet} = -\frac{1}{f}\frac{d_g V_g}{dt}$$

threshold

$$V_{th}^{jet} = \frac{f L_h^{jet}}{2\pi}$$

~ 4 m/s

energy

$$E_i^{jet} = \left(V_a^{jet} - V_{th}^{jet}\right)^2$$

Frontal genesis

Frontal evolution

$$\frac{d_g v_g}{dt} = -f u_a$$

$$\frac{d_g b_g}{dt} = -N^2 w_a$$

→ Cross-frontal circulation (frontogenesis function)

$$N^2 \frac{\partial^2 \psi^{front}}{\partial x^2} + f^2 \frac{\partial^2 \psi^{front}}{\partial z^2} = 2 \frac{g}{\theta_0} F^{front}$$

$$\Rightarrow \psi^{front} \propto \frac{g}{\theta_0 f^2} L_z^{front^2} F^{front}$$

"slaved flow"

$$u_a^{front} \propto \frac{g}{\theta_0 f^2} L_z^{front} F^{front}$$

threshold

$$F_{th}^{front} = \frac{1}{2\pi} \frac{\theta_0 f^2 N}{g}$$

~ 0.2 K / 100 km / h

energy

$$E_i^{front} = \left(\frac{g L_z^{front}}{\theta_0 f^2} \right)^2 \left(F^{front} - F_{th}^{front} \right)^2$$

Convection cells

Latent heating

$$\frac{\partial u_a}{\partial t} = fv_a - \frac{\partial \phi_a}{\partial x}$$

$$\frac{\partial v_a}{\partial t} = -fu_a - \frac{\partial \phi_a}{\partial y}$$

$$\frac{\partial}{\partial t}\frac{\partial \phi_a}{\partial z} = -N^2 w_a + \frac{g}{\theta_0} Q$$

→ Convection cells (latent heating in linear approximation)

$$N^2 \frac{\partial^2 \psi^{conv}}{\partial x^2} + f^2 \frac{\partial^2 \psi^{conv}}{\partial z^2} = \frac{g}{\theta_0}\frac{\partial Q^{conv}}{\partial x}$$

$$\Rightarrow \psi^{conv} \propto L_x^{conv} \frac{Q^{conv}}{\partial \theta_0 / \partial z}$$

"slaved flow"

$$u_a^{conv} \propto \frac{L_x^{conv}}{L_z^{conv}} \frac{Q^{conv}}{\partial \theta_0 / \partial z}$$

threshold

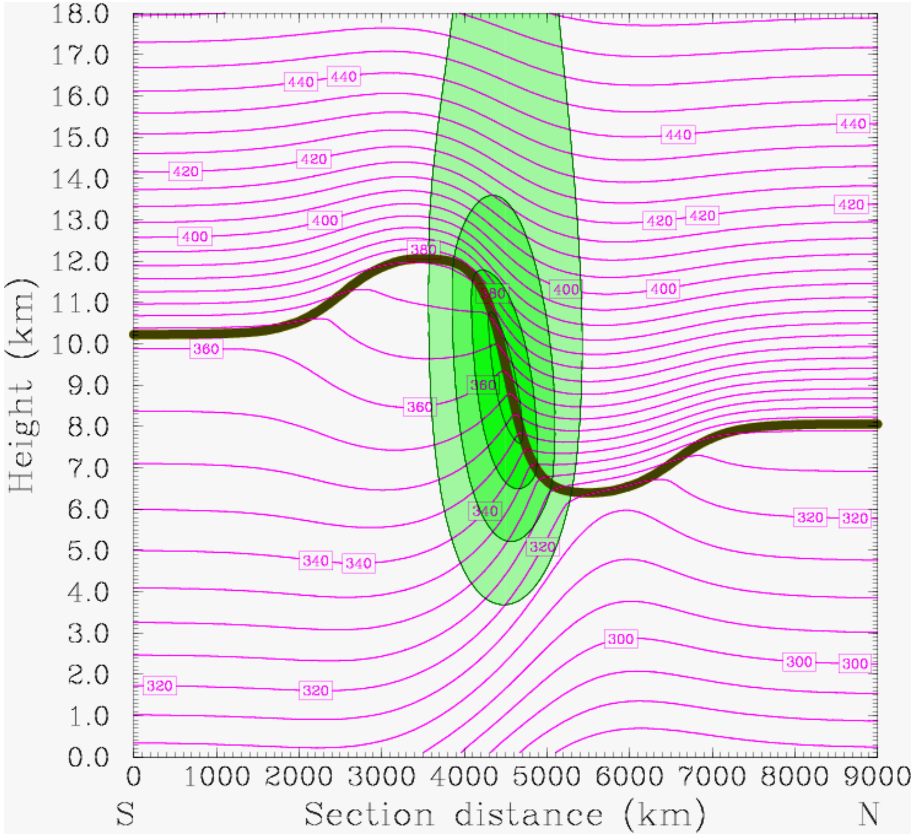
$$Q_{th}^{conv} = \frac{1}{2\pi} \frac{\partial \theta_0}{\partial z} \frac{L_z^{conv}}{f} \sim 0.5\ \mathrm{K/h}$$

energy

$$E_i^{conv} = \left(\frac{1}{\partial \theta_0 / \partial z} \frac{L_z^{conv}}{L_x^{conv}} \right)^2 \left(Q^{conv} - Q_{th}^{conv} \right)^2$$

Simulations

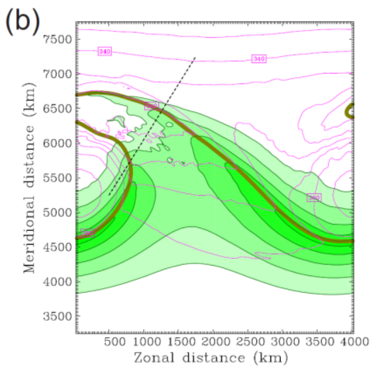
Initial meridional cross section with zonal wind (green), EPV contour (bold) and potential temperature (magenta)



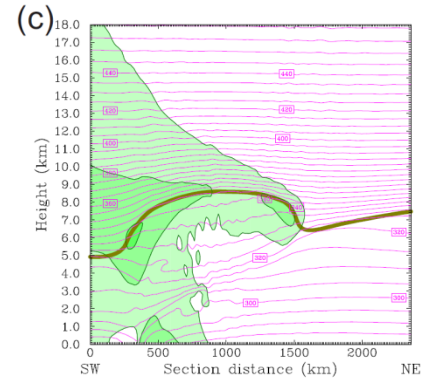
Run codes	RH = 0 %	RH = 40 %	RH = 55 %
Δh = 50 km		LOW	
Δh = 25 km	DRY	MOIST	HUMID
Δh = 12.5 km	HIDRY	HIGH	

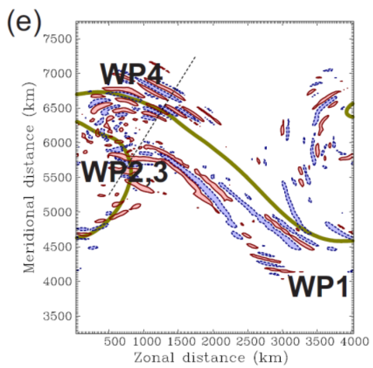
Summary of six 50-day-long simulations with the NCAR Weather Research & Forecasting (WRF) model in a f-plane channel of 4000 km x 10000 km x 26 km size

Structures

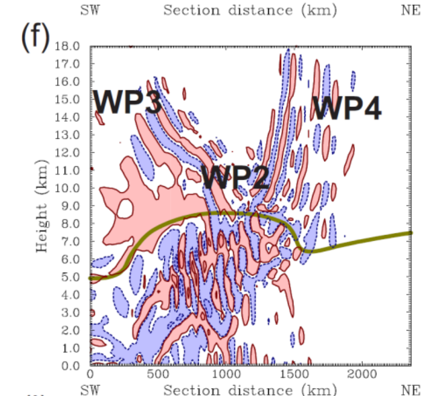


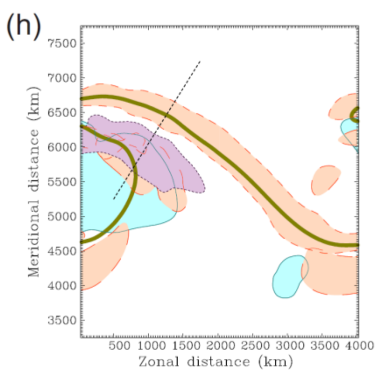
Maps (left) and sections (right) from the MOIST run showing the baroclinic wave with the wind speed (green), potential temperature (magenta) and the 3-PVU Ertel potential vorticity, …



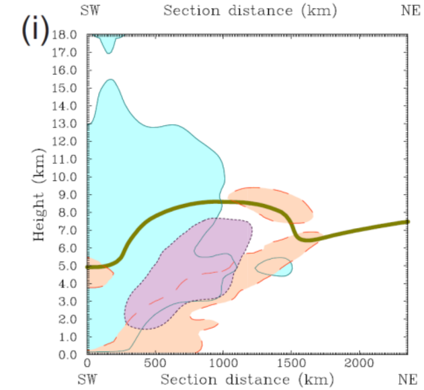


the gravity waves with the horizontal divergence (red/blue) and the 3-PVU line, …





and the related forcing functions with the cross-stream ageostrophic wind (cyan), the dry quasi-geostrophic frontogenesis function (orange) and the latent heating (violet).



➔ During the baroclinic wave life cycle, a variety of gravity waves appeared in accord with intensifying jet and front systems including convective activity.

Energy

The height-integrated gravity wave energy (E_{IGW}) was estimated from a three-dimensional harmonic analysis of the horizontal divergence (Zülicke & Peters, 2006).

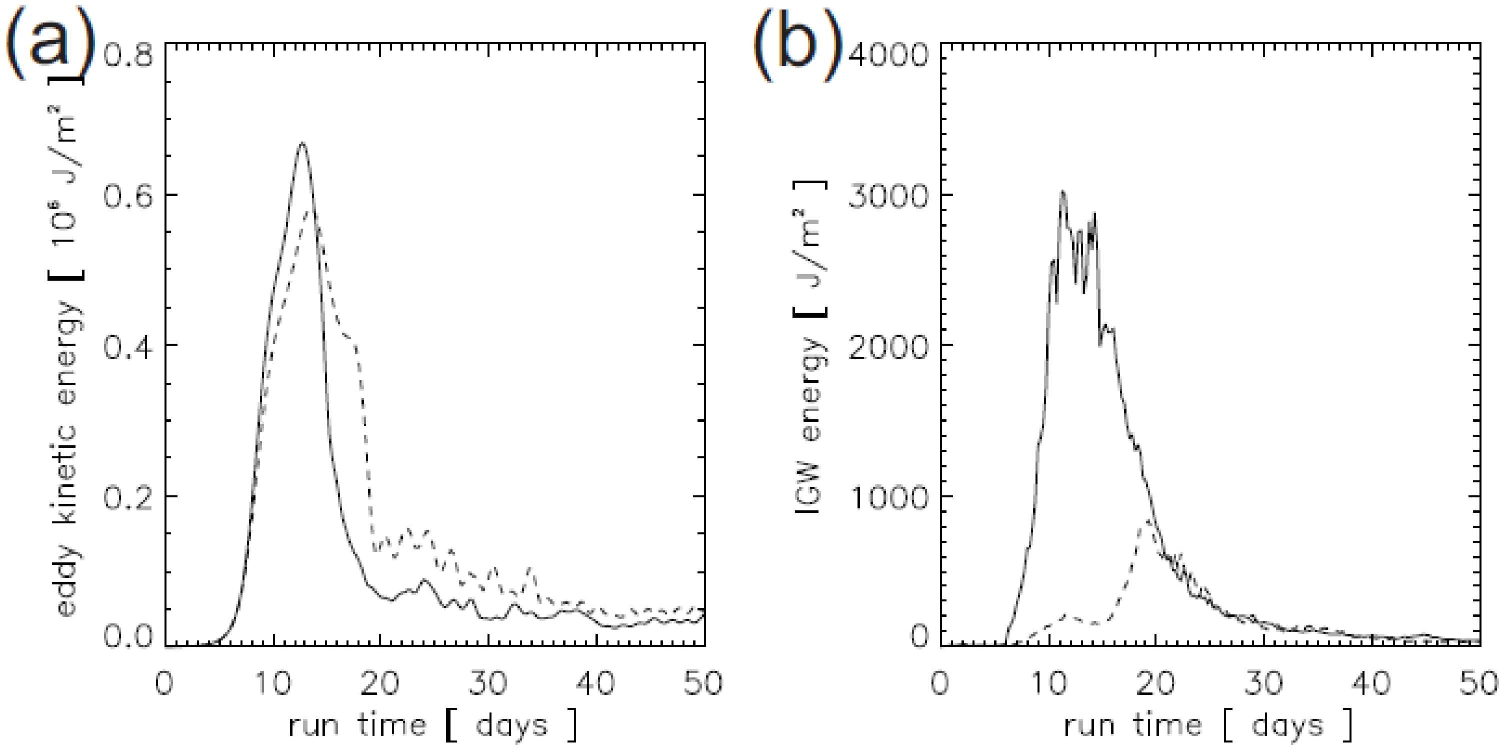


FIG. 9. Time evolution of (a) the eddy kinetic energy (K_E) and (b) the IGW energy (E_{IGW}) integrals for the DRY (dashed) and MOIST (solid) model runs.

➔The ratio of eddy-to-gravity wave energy increased from DRY to MOIST. 2 days delay between tropospheric and stratospheric EIGW is likely due to vertical propagation.

Parameterization

Theoretical results are used to construct a parameterized GW energy :

$$E_{IGW,strato}^{para} = C_{prop} C_{gen} \left(E_{IGW,tropo}^{jet} + E_{IGW,tropo}^{front} + E_{IGW,tropo}^{conv} \right)$$

Parameterization function for the energy of stratospheric IGWs from tropospheric forces. It consists of dimensionless prefactors (for generation and propagation), and source functions in the jet streak (Lagrangean deceleration), front (fontogenesis function) and convection (latent heating), calculated of 500-km horizonally smoothed fields.

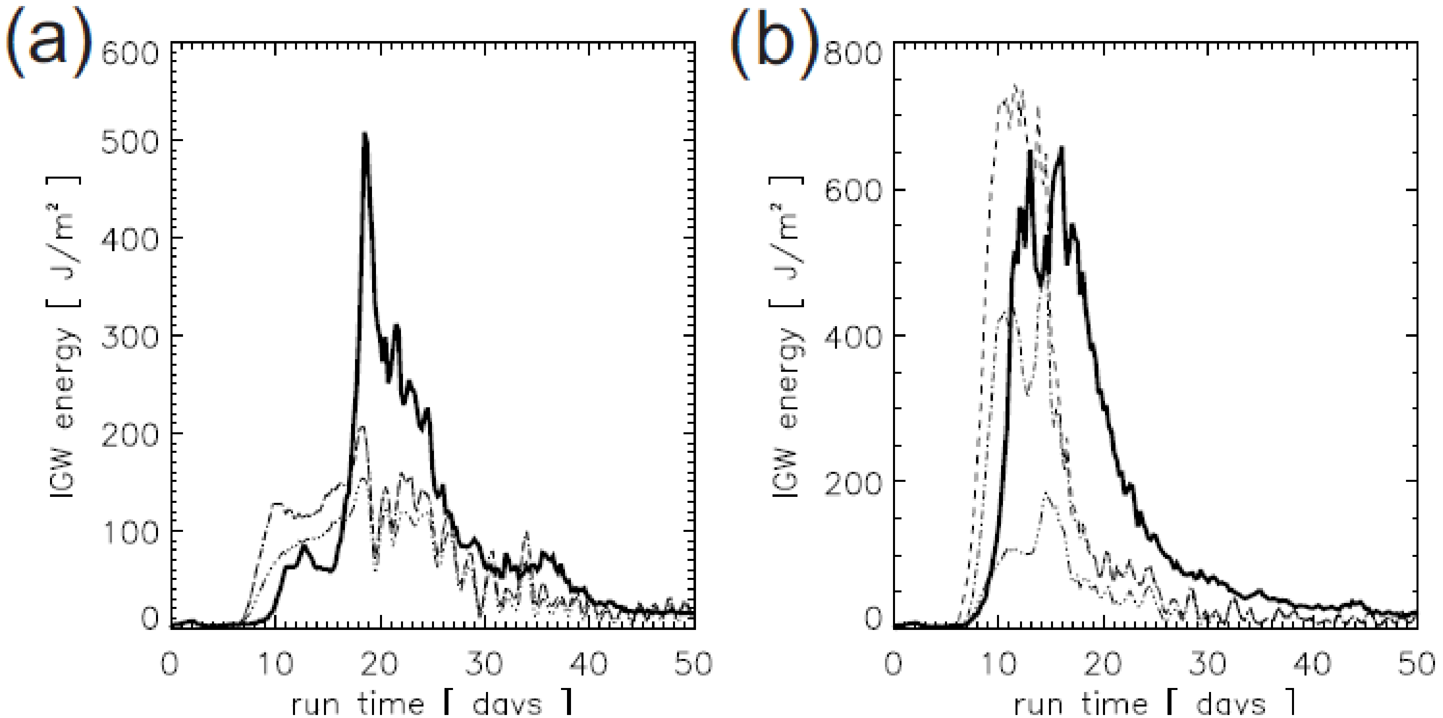
➔ Described IGW radiation from several flow regimes with a unified approach and only one dimensionless parameter (generation prefactor C_{gen}).

➔ The propagation from the troposphere into the stratosphere is included with the other dimensionless parameter (propagation prefactor C_{prop}) depending on the mean wind and temperature.

➔ Characteristic horizontal and vertical scales have been visually estimated and were kept constant for all simulations. The forcing intensities (wind deceleration, frontogenesis function and latent heating) have been taken after 500-km-smoothing.

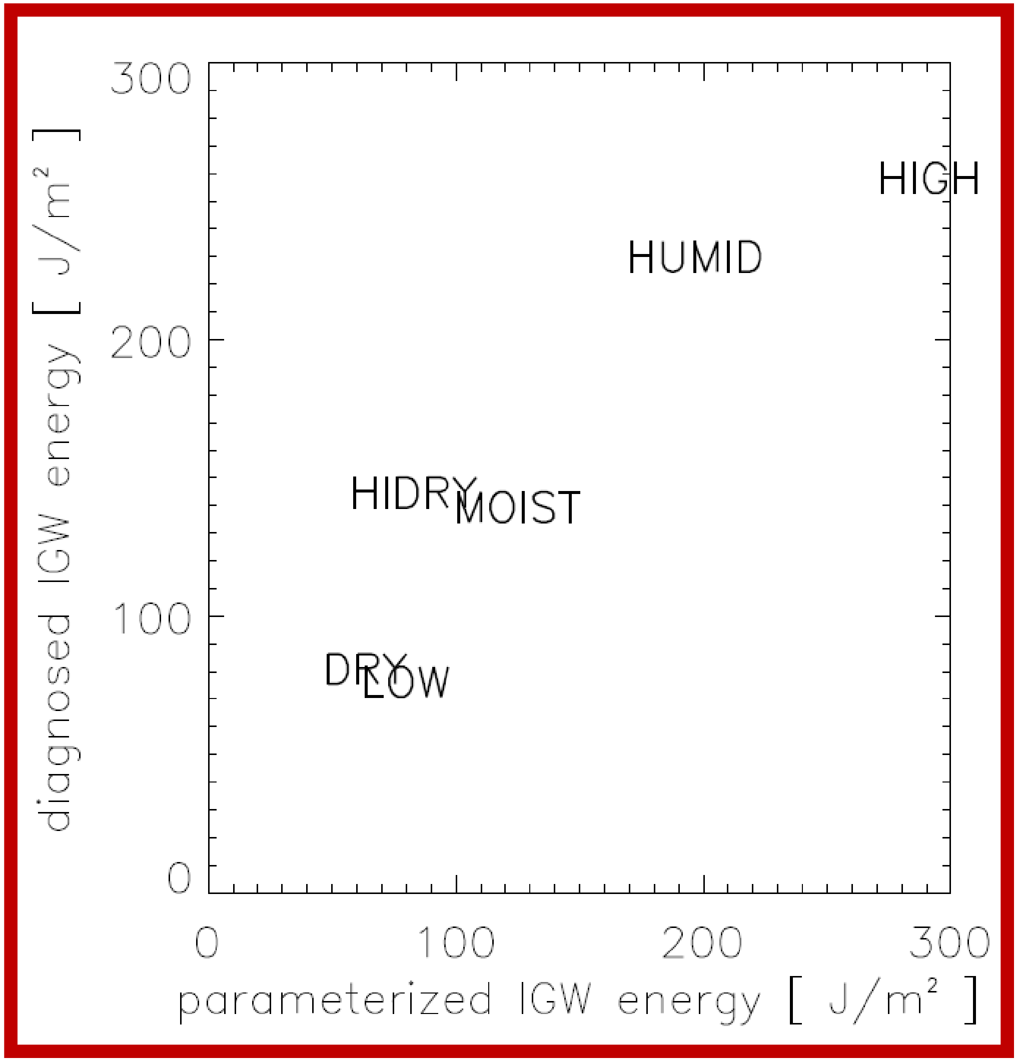
Time series

Compare the diagnosed stratospheric gravity wave energy with the parameterization, constructed from tropospheric forcing functions.



Time evolution of the stratospheric IGW energy for the (a) DRY and (b) MOIST model runs. (Solid: diagnosed IGW energy $E_{IGW,strato}$; dashed: parameterized IGW energy ($E_{IGW,\ strato}^{para} = E_{IGW,\ strato}^{jet} + E_{IGW,\ strato}^{front} + E_{IGW,\ strato}^{conv}$); thin dash-dotted: front-and-jet part; thin dash-dot-dotted: jet part only).

➔The time series showed high energies in gravity waves and source during the overturn phase.



Scatter plot of diagnosed versus parameterized mean stratospheric gravity-wave energy for the six model runs. The propagation prefactor (C_{prop}) is estimated with a simplified WKB model to be about 0.01 which rounds up to the empirically chosen prefactor (C_{para}) of 0.012.

➔ The new non-orographic gravity-wave energy parameterization performed with **93 %** explained variance.

Summary

The generation of gravity waves was related to large-scale faster-than-f modes of the „slaved“ ageostrophic flow covering situations with a jet, front and moist convection. In six WRF simulations, most gravity waves appeared during the overturn phase of the baroclinic wave. The suggested parameterization of gravity wave energy explained about 93 % of the variance.

Conclusion

A theoretical framework was shown to guide the systematic construction of a new non-orographic gravity-wave source parameterization without *tuning constants*. Its performance to deal with space-and-time-dependent flows was demonstrated with simulations of an idealized baroclinic wave.

Studies of straight and curved jets are planned in a new project „Spontaneous Imbalance“ which is part of the research unit →

MS-GWaves

DFG Deutsche Forschungsgemeinschaft

Publication

Mirzaei, M., C. Zülicke, A. R. Mohebalhojeh, F. Ahmadi-Givi & R. Plougonven, 2014: *J. Atmos. Sci.* **71**: 2390 - 2414. doi:10.1175/JAS-D-13-075.1.

IAP-SI: Project work to complete an explicit IGW parameterization…

Theory

Extend to include wavenumbers and the radial deceleration

- WP-IAP1 (Student preparation and diagnostics) ↔ SI(GUF,BTU)
- WP-IAP2 (Theoretical studies for parameterization) ↔ SI(GUF)

Validation

Simulate a variety of flow regimes with straight and curved jets

- WP-IAP3 (Idealized WRF simulations)
- WP-IAP5 (Validation of parameterization with WRF simulations)

$$\boldsymbol{E}, k, l, m, \omega$$

Energy, wavenumbers and frequency as elements of the parameterization

Idealized straight and curved jets to be simulated for validation

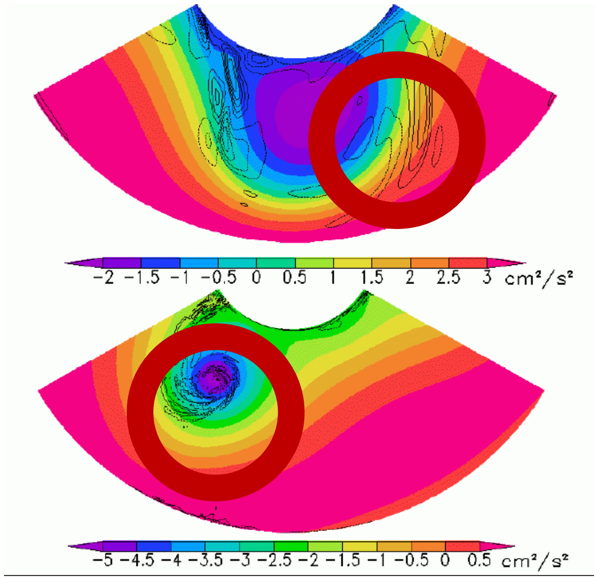
IAP-SI: … and its application

Applications

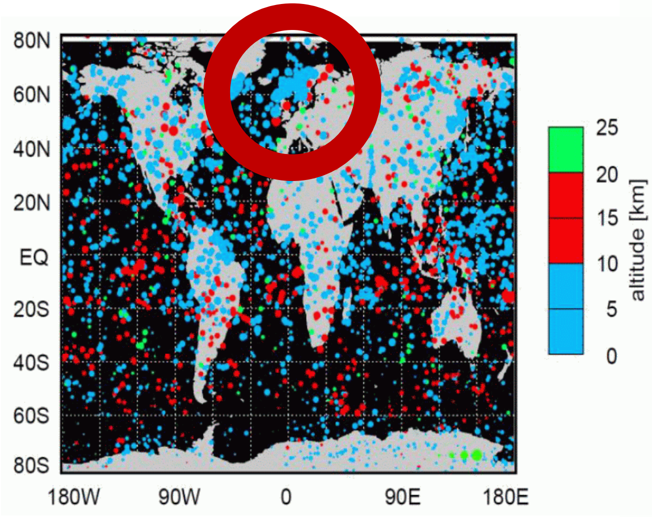
Use the parameterization for the diagnosis of SI sources

- WP-IAP4 (Validation with rotating annulus experiment) ↔ SI(GUF,BTU)
- WP-IAP6 (Application in a global context) ↔ SV(FZJ), GWING(MPI,DWD)

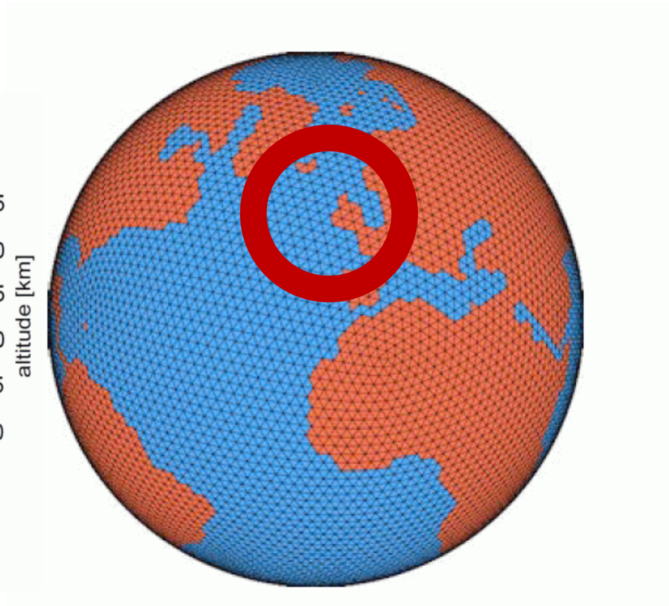
and, in the second project phase, for the prediction of gravity wave traces.



Laboratory experiments with the rotating annulus by SI(BTU, GUF)



Global satellite observations & ray tracing experiments by SV(FZJ)



Global simulations with MA-ICON by GWING(MPI,DWD)