A new theory for downslope windstorms and trapped lee wave

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1)Motivation: trapped lee waves

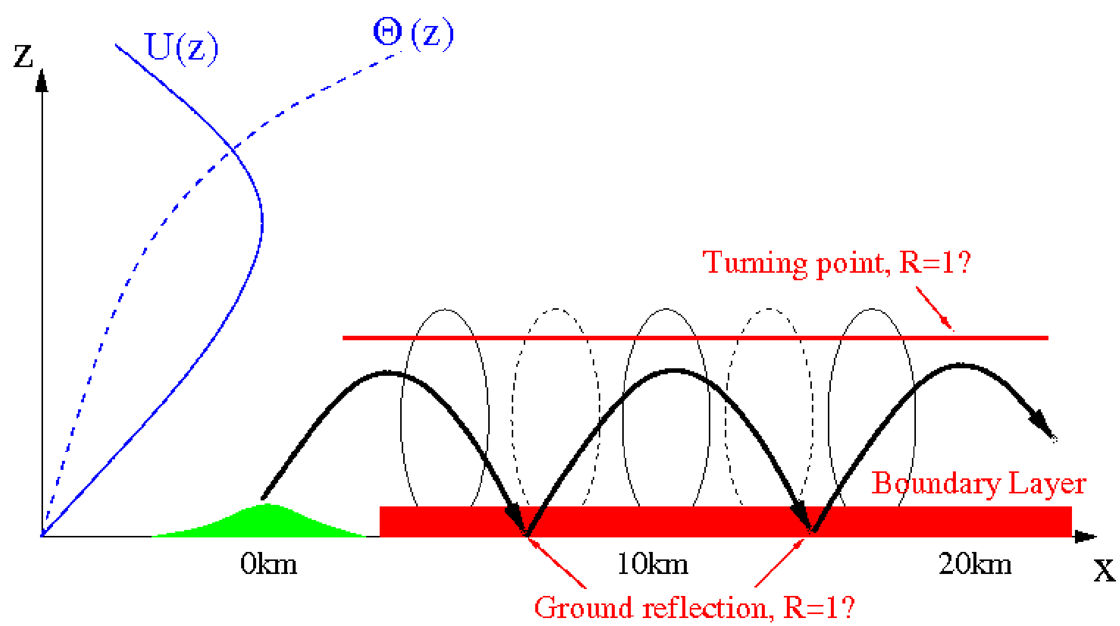
2)Model description

3)Downslope windstorms and Foehn

4)Trapped lee-waves and low level flow stability

Downslope windstorms and trapped lee wave, F. Lott
1) Motivation: trapped lee waves

Gravity waves trapping and lee waves (Scorer 1949)



Turning points are not sufficient for the existence of trapped lee-waves, $|R|$ matters (Smith et al. 2002) and Jiang et al. 2006):

Often $|R|$ is small ($|R|<<1$), except when the boundary layer is unstable!

A good reason for $|R|<<1$: the background wind is null in $z=0$, there is a critical level for mountain waves !

The gravity waves somehow see the presence of this critical level (Lott 2007)

Downslope windstorms and trapped lee wave, F. Lott

1) Motivation: trapped lee waves

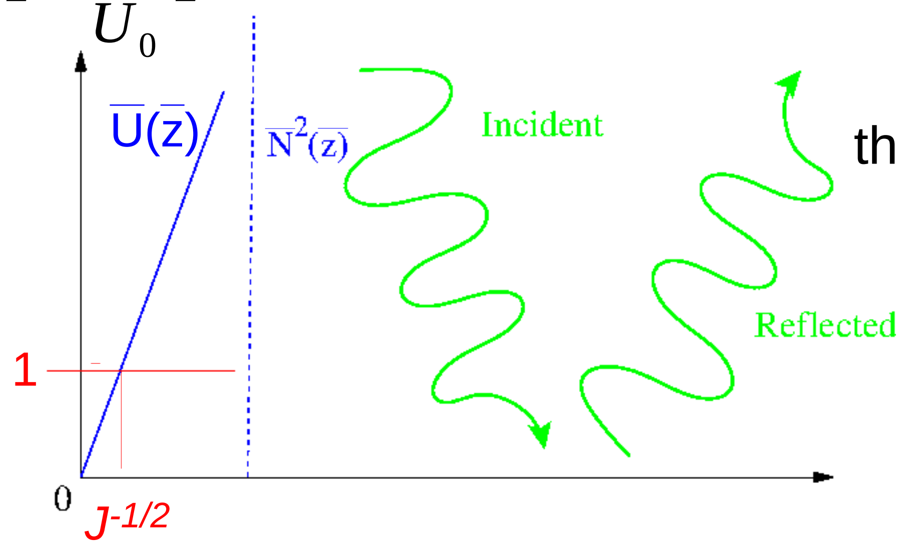
For a viscous bound. |R| depends on the surface Richardson number J

$|R|\sim 1$ can only occur when $J<0.25$ and in the inviscid limit (Lott 2007)

Case of a small linear damping ε: J>0.25

$w=\hat{w}\,e^{ikx}$ Monochromatic GW

$\bar{z}=\frac{N_0}{U_0}z$



Near the surface (Booker and Bretherton 1967):

$$\hat{w}\approx a_1(z-i\epsilon)^{1/2-i\sqrt{J-\frac{1}{4}}}+a_2(z-i\epsilon)^{1/2+i\sqrt{J-\frac{1}{4}}}$$

the boundary condition: $w(z=0)=0$ yields

$$|R|=\left|\frac{a_2}{a_1}\right|=\left|(-i\epsilon)^{i\sqrt{J-0.25}}\right|=e^{-\pi\sqrt{J-0.25}}$$

It decreases when J increases.

We will return to the unstable case

J<0.25 latter

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2) Model description

How to integrate a mountain forcing in those results
(the linear boundary condition w(0)=U(0)dh/dx reduces to 0=0)!

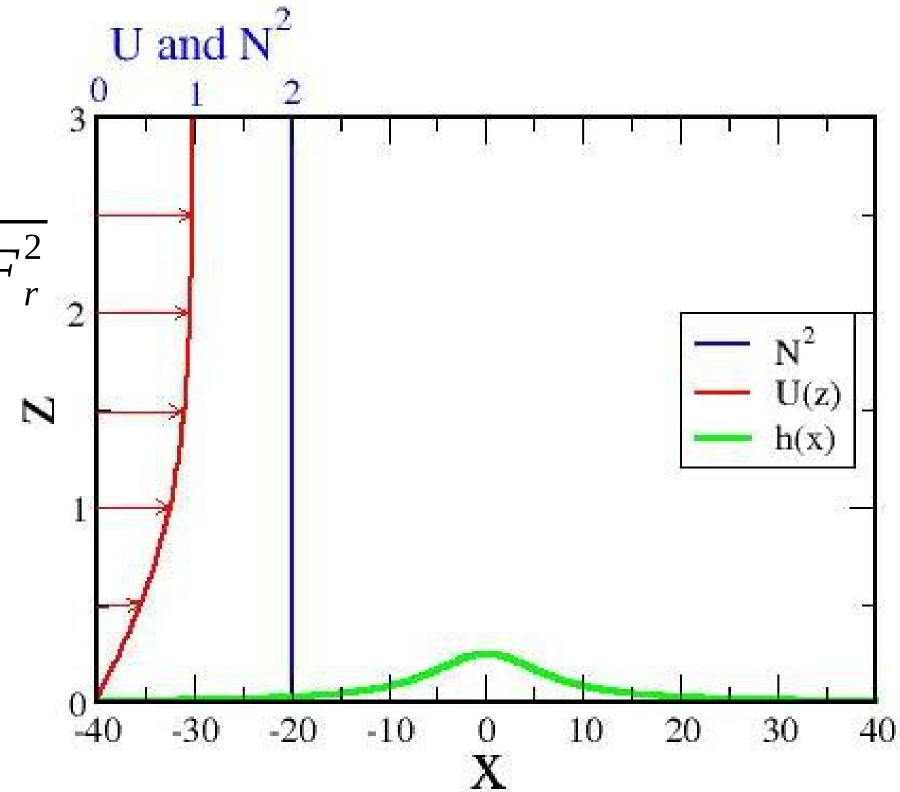
2D Boussinesq non-dimensional

$$U=\tanh z/\sqrt{J}\,,\ N^2=1 \qquad \text{Mountain}: h=\frac{H_n}{1+x^2/F_r^2}$$

$$\sqrt{J}=\frac{d\,N}{U_{0,}} \quad \text{non-dim shear depth}$$

$$H_N=\frac{H\,N}{U_0}\ , \quad \text{non dim mount. height}$$

$$F_r=\frac{L\,N}{U_0}\,,\text{Froude or non dim mountain length}$$



Linear inflow solution:

$$w(x,z)=\int_{-\infty}^{+\infty} f(k)\,\hat{w}_c(k,z)\,e^{ikx}\,dk$$

Nonlinear boundary condition:

$$w(h)=U(h)\frac{dh}{dx}$$

~Long (1953)

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~Long (1953)

$\hat{w}_c(k,z)$ satisfies:

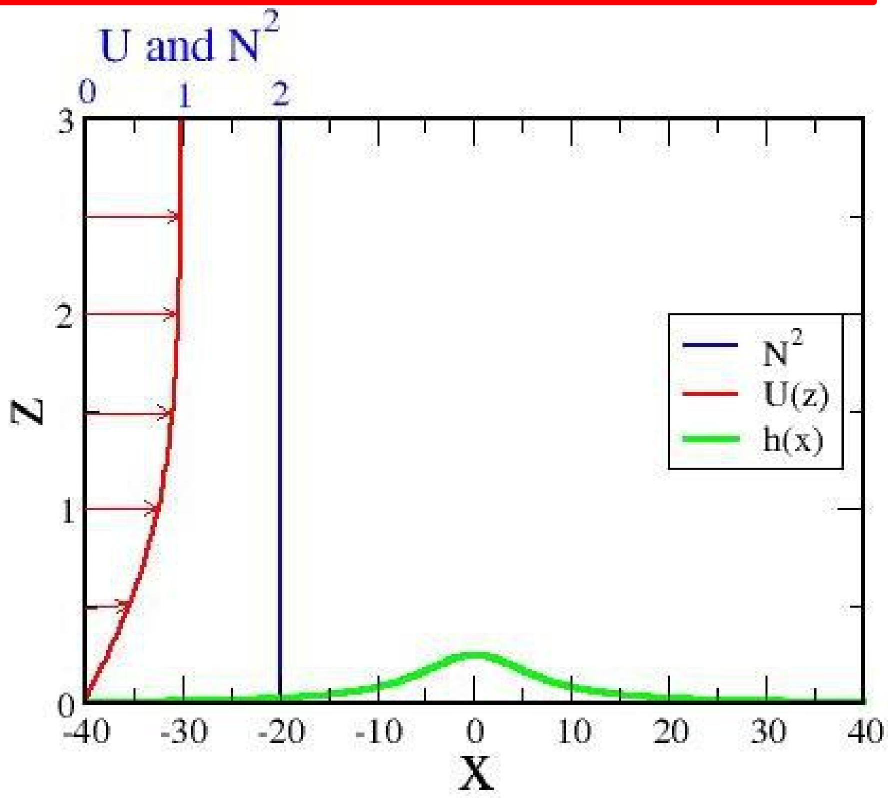
$$\frac{d\hat{w}_c}{dz}+\left(\frac{1}{U^2}-2\frac{1-U^2}{J}-k^2\right)\hat{w}_c=0$$

which has an “analytical” solution, satisfying

$$\hat{w}_c\approx e^{-mz/\sqrt{J}} \qquad \text{when } z\to\infty$$

$$\hat{w}_c\approx a_1(k)z^{1/2-i\mu}+a_2(k)z^{1/2+i\mu} \qquad \text{when } z\to 0$$

$$m^2=k^2-1 \text{ and } \mu^2=J-0.25$$



Nonlinear boundary condition determine *f(k)*:

$$\int_{-\infty}^{+\infty} f(k)\left(a_1(k)h(x)^{1/2-i\mu}+a_2(k)h(x)^{1/2+i\mu}\right)e^{ikx}\,dk=h\frac{dh}{dx}$$

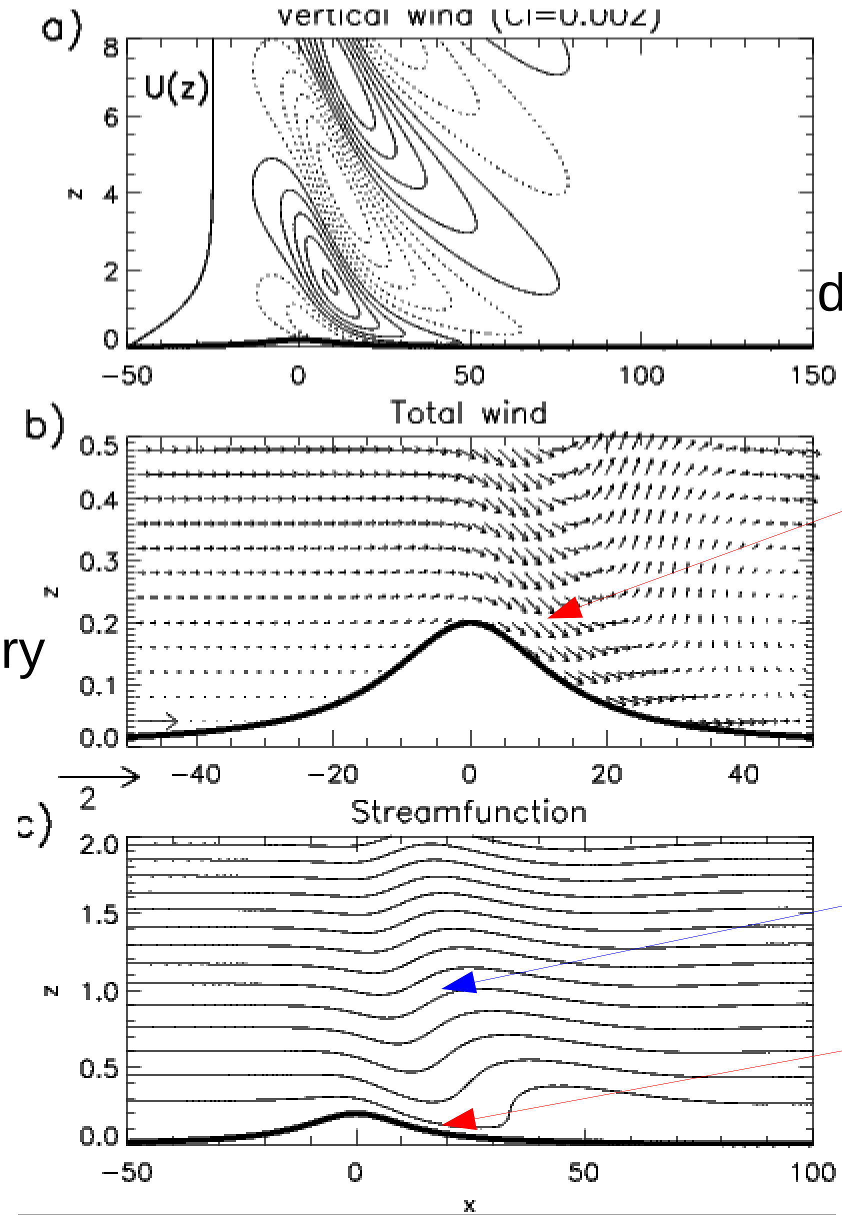
This inversion can be done when there is dissipation $\epsilon\neq 0$

Downslope windstorms and trapped lee wave, F. Lott

3) Prediction of dowslope windstorms and Foehn

w(x,z): structure of a mountain wave with vertical wavelength becoming small near the ground

Results for $J=2,\ H_{ND}=0.2,\ F_r=10$



No trapped waves, despite the fact that

$$S(z)=\frac{1}{U^2}-2\,\frac{1-U^2}{J}$$

decreases with altitude

Remember

$$\hat{w} \underset{z\to 0}{\approx} z^{1/2}\, e^{+i\sqrt{J-0.25}\log z}$$

This also means that the horizontal wind becomes very Large near the surface

$$\hat{u} \underset{z\to 0}{\approx} z^{-1/2}\, e^{+i\sqrt{J-0.25}\log z}$$

This leads to the strong downslope winds

Strong downslope winds

No upper level wave breaking

Dry Foehn

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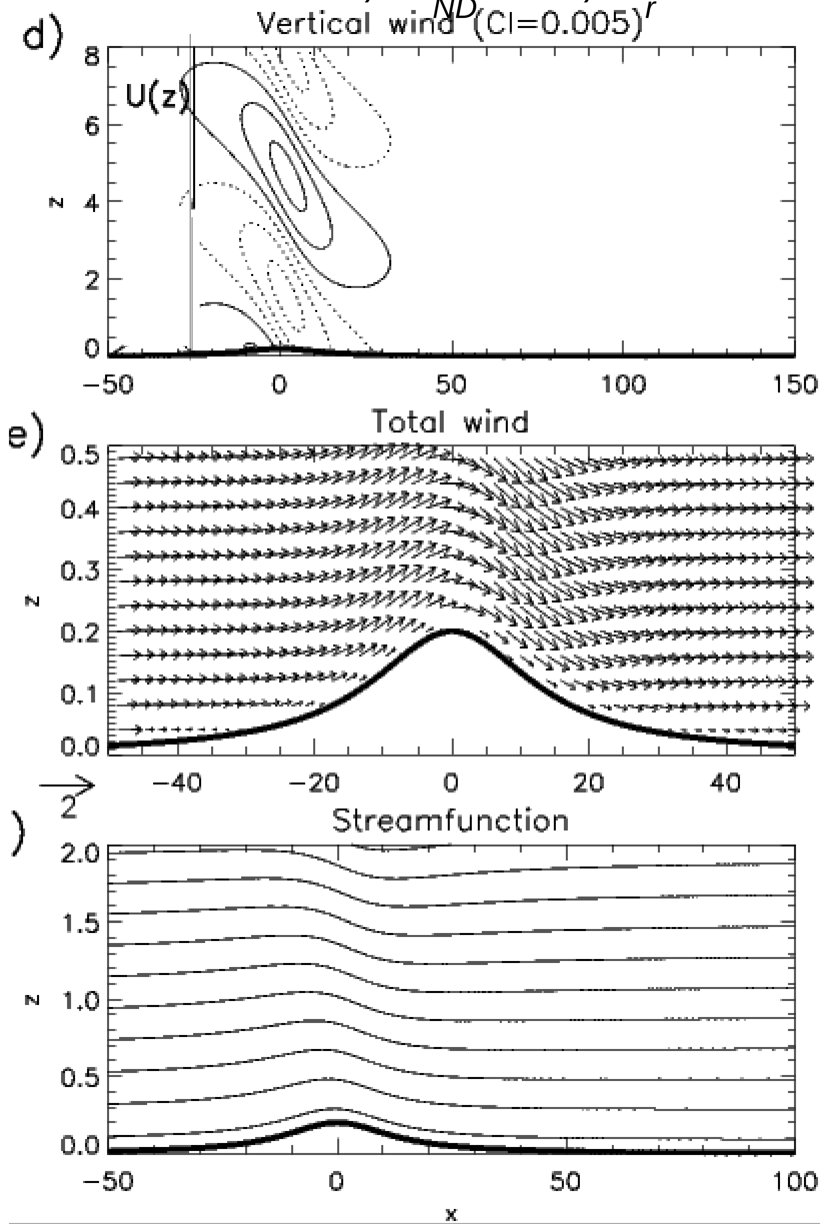
3) Prediction of dowslope windstorms and Foehn

Uniform wind, H_{ND}=0.2, F_r=10



These results essentially Result from the smallness of the background horizontal wind near z=0

Here simulations with *U=1*

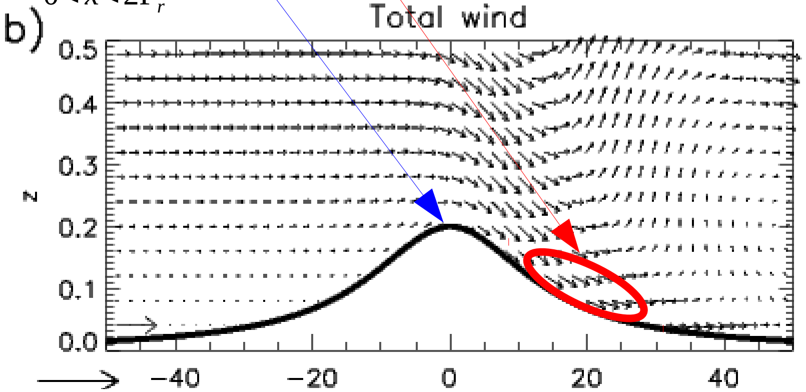
No strong downslope winds or jumps when *U=1*

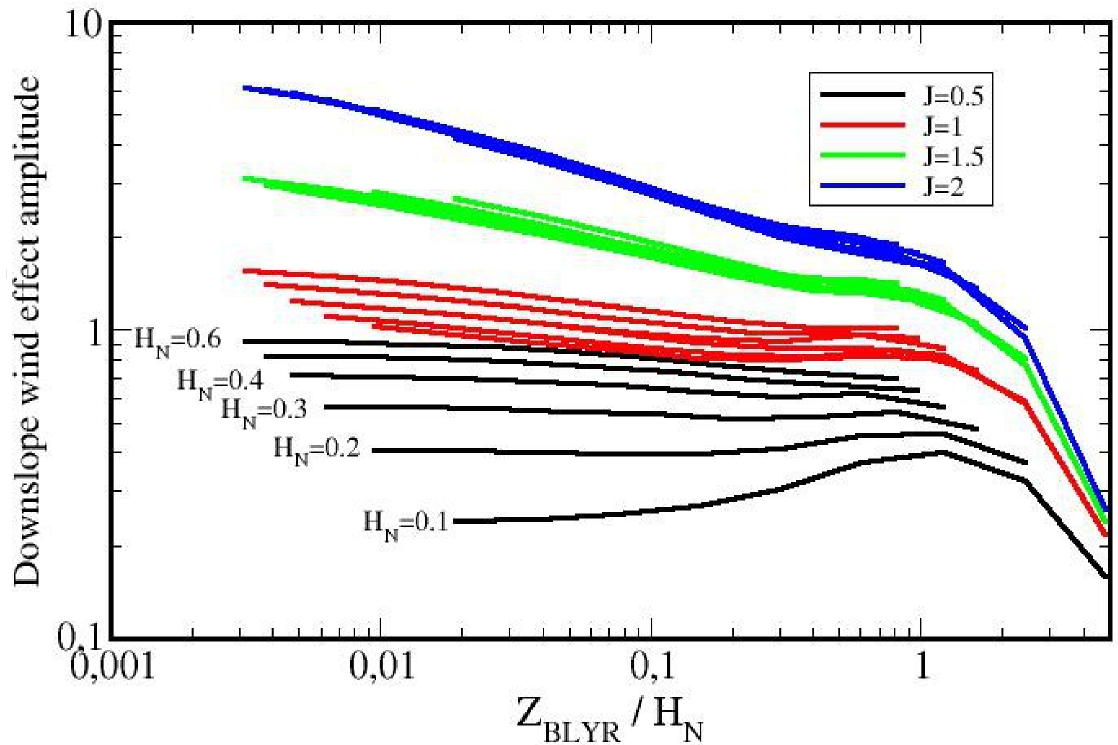
Downslope windstorms and trapped lee wave, F. Lott

3) Prediction of dowslope windstorms and Foehn

Downslope wind amplitude:

$$\underset{\substack{z<H_n/2 \\ 0<x<2F_r}}{\mathrm{Max}}\left(\frac{u(x,z)}{U(H_N)}\right)$$





Boundary layer depth:

$$Z_{BLYR} \approx 5\epsilon / F_r$$

ε being the linear damping parameter

Downslope windstorms
and “dry” Foehn
are favored in stable flows

Downslope windstorms and trapped lee wave, F. Lott

4) Trapped lee waves and low level flow stability

$|R|\sim 1$ can only occur when $J<0.25$ and in the inviscid limit (Lott 2007)

Near the surface:

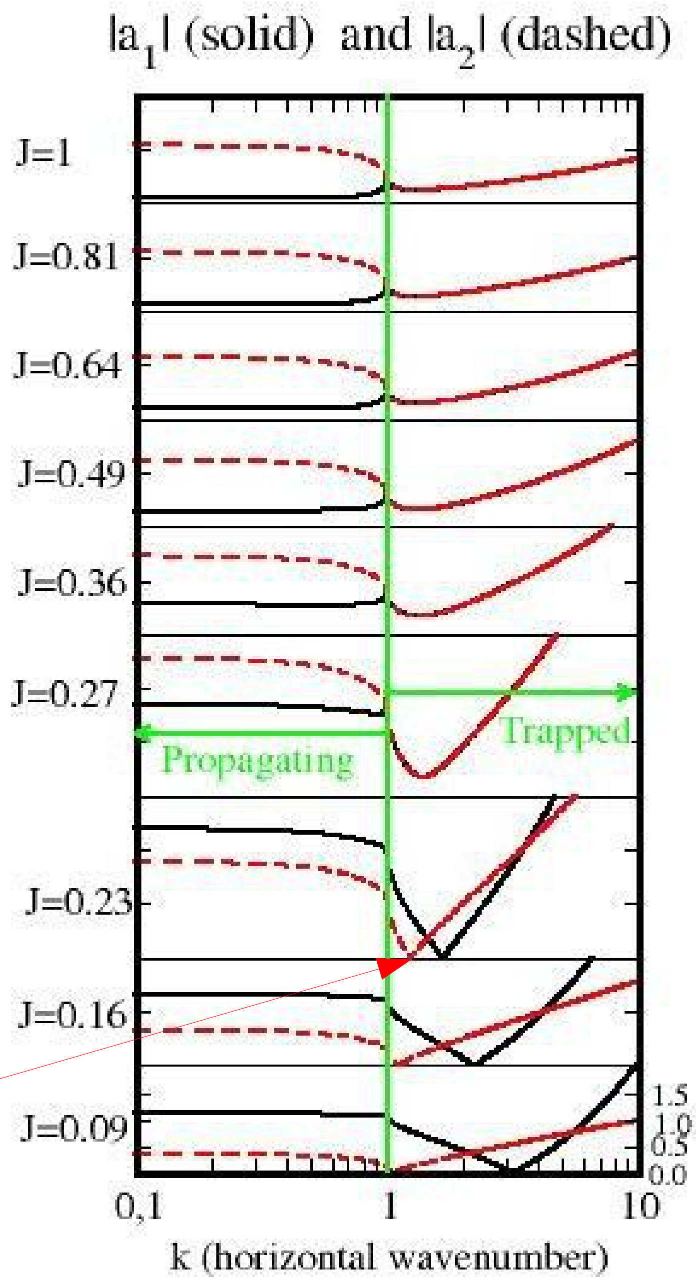
$$\hat{w} \approx a_1 (z-i\epsilon)^{1/2+\sqrt{\frac{1}{4}-J}} + a_2 (z-i\epsilon)^{1/2-\sqrt{\frac{1}{4}-J}}$$

The BC: $w(z=0)=0$ yields $\left|\frac{a_2}{a_1}\right| = \epsilon^{\sqrt{0.25-J}}$

The EP flux $\frac{\hat{u}\,\hat{w}^*}{2} = \frac{\sqrt{0.25-J}}{2k} |a_1|^2 \epsilon^{\sqrt{0.25-J}}$

goes to zero when ε goes to zero: $|R| \rightarrow 1$

Also, modes with $a_2=0$ exists in our profiles: the longest neutral modes of KH instability (Drazin 1958)



4) Trapped lee waves and low level flow stability

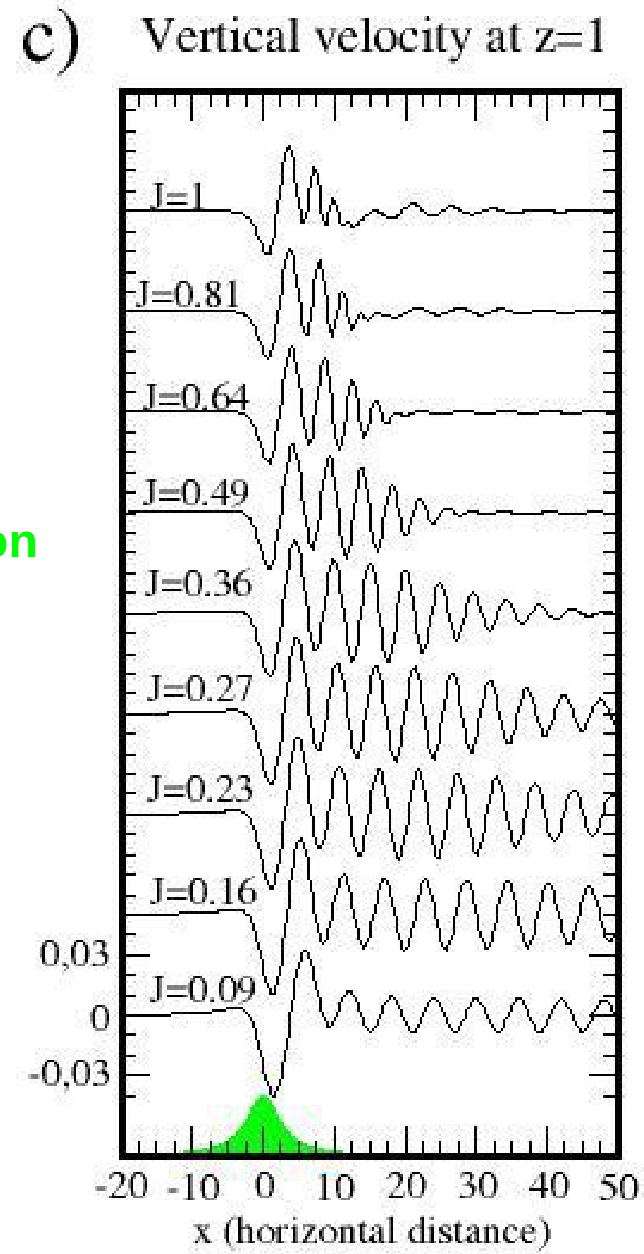
Stable (J<0.25), waves stay confined over the ridge

Trapped waves decaying downstream

Transition

Unstable (J<0.25), pure trapped lee waves develop downstream

Pure trapped waves= longest neutral mode of KH instability (Drazin 1958)



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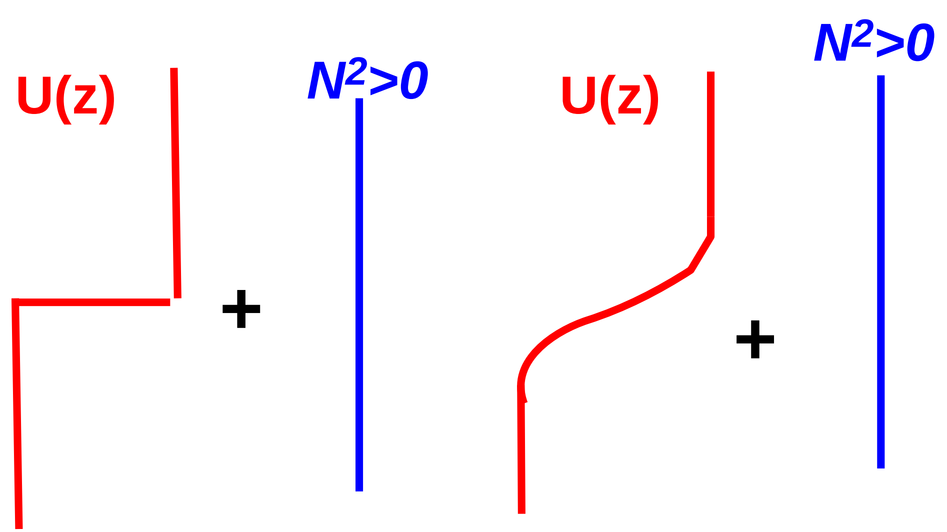
Critical level dynamics induces downslope windstorms and Foehn when J>1

Trapped lee waves start to appear when J<1

Pure trapped lee waves when J<0.25 are near KH instability

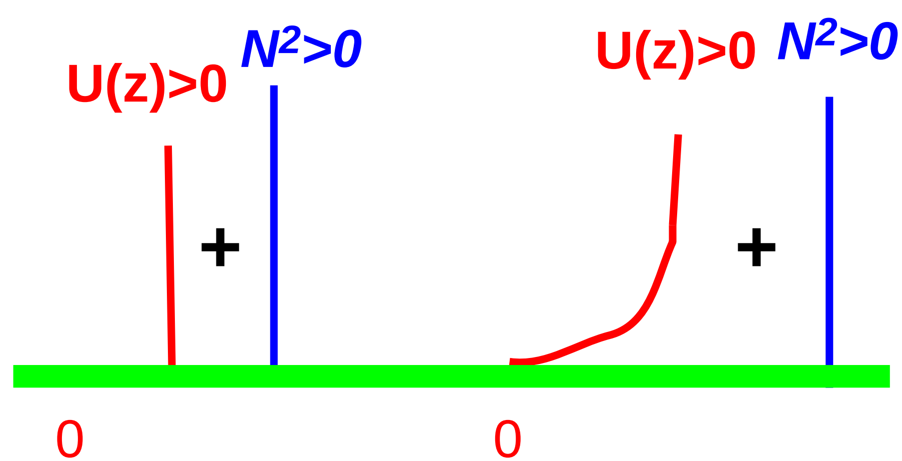
KH instabilities:



Always Unstable (actually J=0)

Conditionnaly Unstable (if J<0.25)

Trapped mountain waves:



|R|=1 always

|R|=1 becomes conditional to J<0.25

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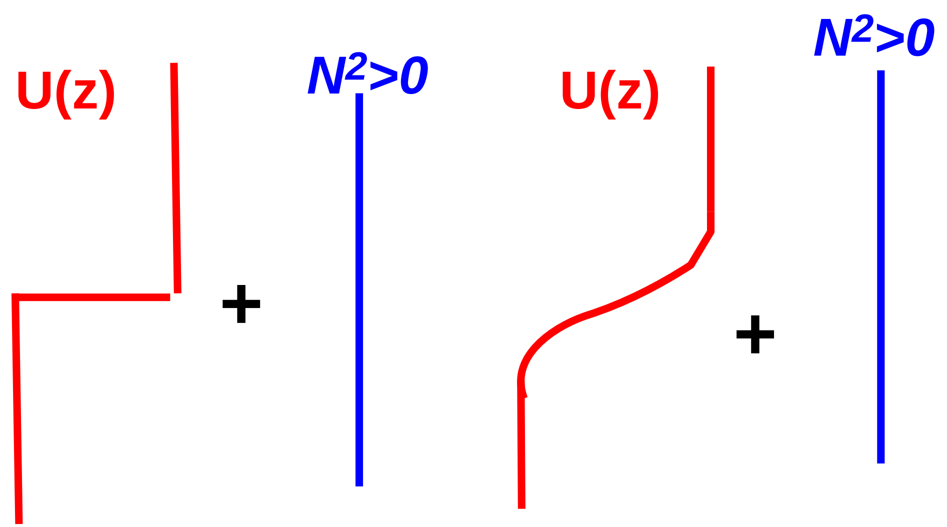

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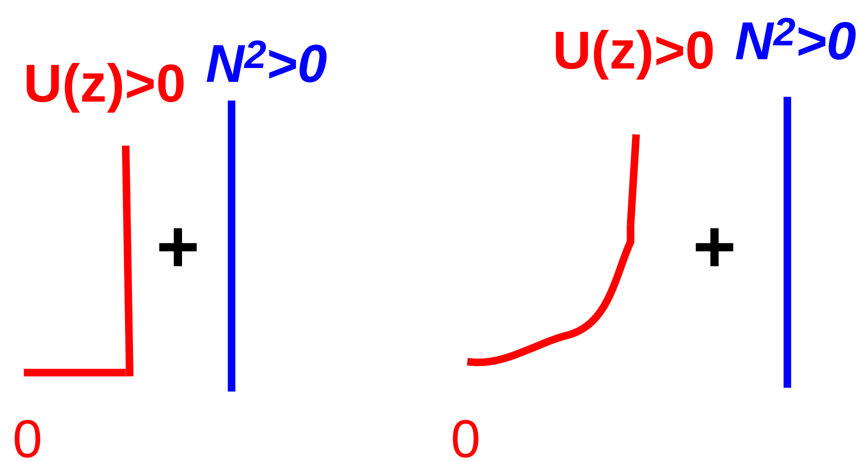
<u>Trapped mountain waves:</u>



|R|=1 always (actually J=0 At the surface)

|R|=1 becomes conditional to J<0.25