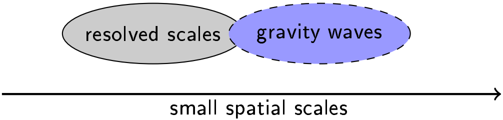
On the numerical implementation of a fully coupled gravity-wave ray tracer in atmospheric models

Gergely Bölöni, Bruno Ribstein, Ulrich Achatz, Jewgenija Muraschko, Christine Sgoff, Junhong Wei

Motivation



- GWs are not fully resolved by GCMs and NWP models ⇒ parametrization ⇒ (Wentzel–Kramers–Brillouin) WKB theory
- Currently used parametrizations: *equilibrium profile* of the wave field, *steady state* background wind, *fully linear theory* ⇒ the resolved flow (mean flow) does not know about the GWs propagating through it ⇒ the GW momentum is deposited *instantenously* to the mean flow at height of wave breaking
- Proposal for improvement: *weakly nonlinear coupling* between the waves and the resolved flow ⇒ continuous interaction between the wave and the mean flow during the propagation

WKB theory

Wave resolving equations (2-D Euler equations, no rotation):

$$\begin{aligned}
\frac{Du}{Dt} + c_p\theta\frac{\partial\pi}{\partial x} &= 0\\
\frac{Dw}{Dt} + c_p\theta\frac{\partial\pi}{\partial z} + g &= 0\\
\frac{D\theta}{Dt} &= 0\\
\frac{D\pi}{Dt} + \frac{\kappa}{1-\kappa}\pi\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right) &= 0
\end{aligned}$$

with	$\frac{D}{Dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + w\frac{\partial}{\partial z}$
Exner pressure	$\pi = (p/p_0)^\kappa$
Pot. temperature	$\theta = T(p_0/p)^\kappa = T/\pi$
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  with $Z = \epsilon z, T = \epsilon t$, $m = \partial\phi/\partial Z$ and $\omega = -\partial\phi/\partial T$
- Scaling for the gravity waves: $\epsilon = \mathrm{L}_w/H_\theta << 1$: weak stratification

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Mean flow

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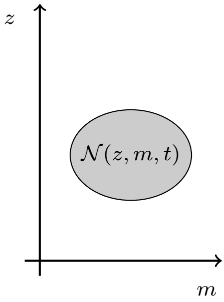
- *Problem*: if rays crossing $\Rightarrow$ *caustics*: several m at same height z $\Rightarrow$ e.g. c_{gz} multivalued BUT! $\mathcal{A} = \mathcal{A}(z,t)$ $\Rightarrow$ wave action conservation ill-defined $\Rightarrow$ numerical problems

WKB theory in phase space

- Solution: extension of the model to a 2D phase space (z, m)



Hertzog et al., 2002,
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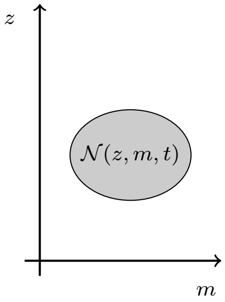

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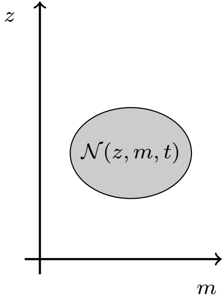

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- Eulerian view

$$\frac{\partial \mathcal{N}}{\partial t} + \frac{\partial (c_{gz}\mathcal{N})}{\partial z} + \frac{\partial (\dot{m}\mathcal{N})}{\partial m} = 0$$

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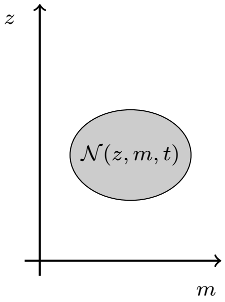

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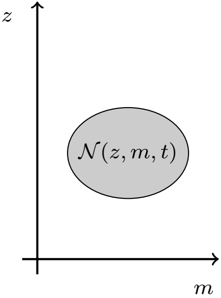

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- *Lagrangian view*

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- Coupled wave - meanflow equations in phase space:

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- extension to *Muraschko et al.*, 2015: *isothermal background with variable density* $\overline{\rho} = \rho(z) = \rho_0 e^{-z/H} \quad H = \frac{RT_0}{g}$

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- If the saturation criterion is fulfilled the wave action density $\mathcal{N}$ *is reset* layer-wise to a value that sets back *stability*

Numerical experiments

Methodology

- LES: fully non-linear wave resolving reference (PincFloit, *Rieper et al.*, 2013)
- WKB-eu: Eulerian WKB model
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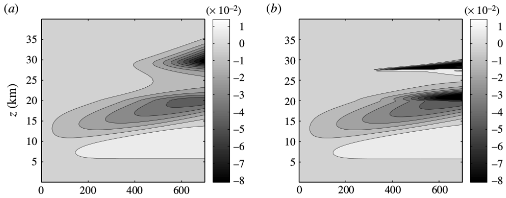
Cases

- refraction by a jet: U_{jet} weak $\Rightarrow m$ only slightly modified
- reflection from a jet: $U_{jet} \geq \frac{N}{k}\left(1 - \frac{k}{\sqrt{k^2+m^2}}\right) \Rightarrow m$ and c_{gz} changes sign
- static instability: amplitude growing until static instability
- modulational instability: $|m| \approx |k| \Rightarrow$ wave packet is shrinking, its amplitude growing $\rightarrow$ static instability
- critical layer: $U_{jet} \approx -c_p \Rightarrow m$ grows to infinity, wavepacket collapses

Numerical experiments

Induced wind, refraction by a jet ($\lambda_x = 10km, \lambda_z = 1km$)



Rieper et al., 2013

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b): conventional WKB raytracer

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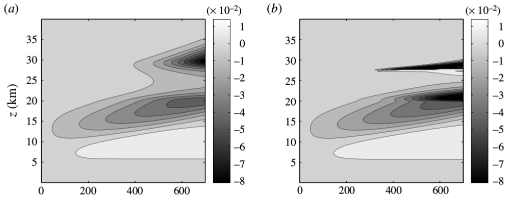

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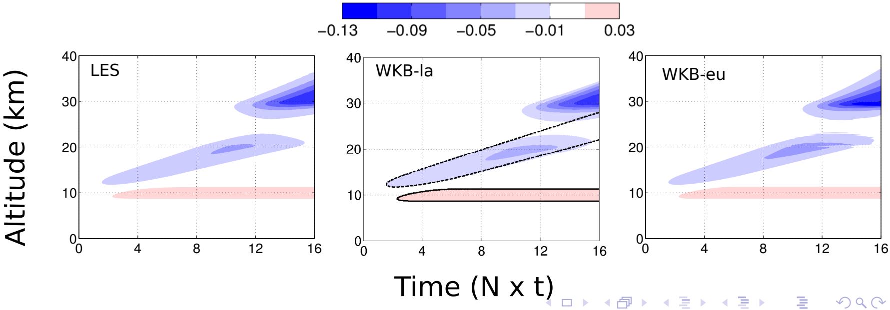

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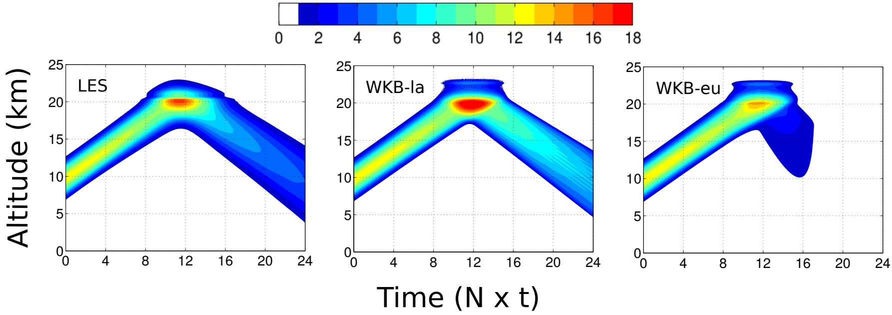
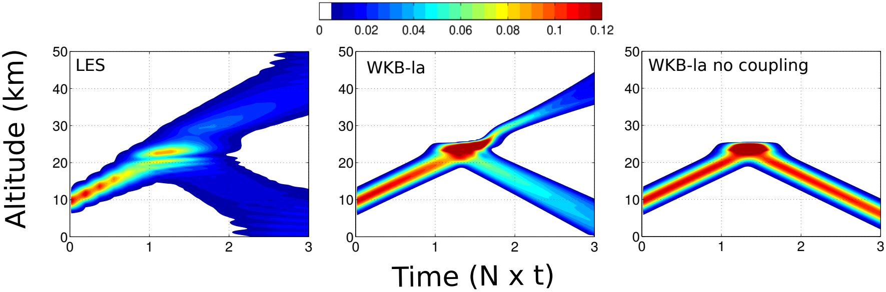
Wave energy, reflection from a jet ($\lambda_x = 10km, \lambda_z = 1km$)



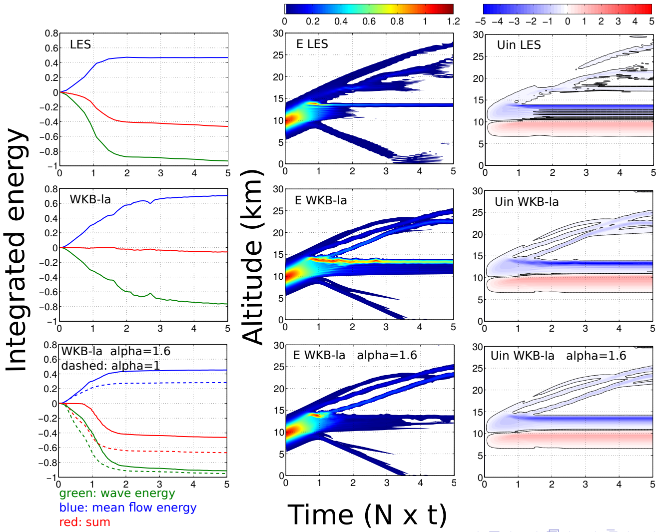
Numerical experiments

Wave energy, partial reflection from a jet ($\lambda_x = 6km, \lambda_z = 3km$)



Numerical experiments

Static instability $(\lambda_x = \lambda_z = 1km)$



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- **The Lagrangian WKB model has been supplemented with a wave breaking parametrization**, which can slightly improve the fit to LES.
- **However! the WKB model without the breaking parametrization works surprisingly well** $\rightarrow$ the most important part of the wave - background interaction is the weakly-nonlinear coupling and *NOT* wave breaking or wave-wave interactions

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- **Caustics problem resolved**: comparison with LES shows that the WKB phase-space concept is working well **in** an **isothermal atmosphere like background** too.
- **The Lagrangian raytracer is very efficient**: factor of 10-100 compared to the Eulerian model and factor of 1000-10000 compared to LES $\Rightarrow$ the Lagrangian model is the main candidate for future work
- **The Lagrangian WKB model has been supplemented with a wave breaking parametrization**, which can slightly improve the fit to LES.
- **However! the WKB model without the breaking parametrization works surprisingly well** $\rightarrow$ the most important part of the wave - background interaction is the weakly-nonlinear coupling and *NOT* wave breaking or wave-wave interactions
- **There is a good reason to try out fully coupled transient approaches** in GW parametrizations and go beyond the non-acceleration theorem applied in most current schemes

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Questions?

Experiment	**Wavepacket**	**Background**	**Domain size**	**Resolution**
REFR Refraction by a jet	Cosine shape $\lambda_x = 10km, \lambda_z = 1km$ $k = 2\pi/\lambda_x, m = 2\pi/\lambda_z$ $z_0 = 10km, \Delta_{wp} = 10km$ $branch = -1, a_0 = 0.1$	non-Boussinesq $T = 300K$ $N \approx 0.018$ $u_0 = 5m/s$ $z_u = 25km$ $\Delta_u = 10km$	WKB Euler: $L_z = 40km$ $m \in [0.001, 0.008]$ WKB Lagrange: $L_z = 40km$ LES: $L_z = 40km, L_x = 10km$	WKB Euler: $nz = 400, nm = 70$ $dz \approx 100m, dm = 10^{-4}s^{-1}$ WKB Lagrange: $nz = 400, dz_{smooth} \approx 600m$ $dz \approx 100m, n_{ray} = 4000$ LES: $nz = 1280, nx = 32$ $dz \approx 31m, dx = 310m$
REFL Reflection from a jet	Cosine shape $\lambda_x = 10km, \lambda_z = 1km$ $k = 2\pi/\lambda_x, m = 2\pi/\lambda_z$ $z_0 = 10km, \Delta_{wp} = 10km$ $branch = -1, a_0 = 0.1$	non-Boussinesq $T = 300K$ $N \approx 0.018$ $u_0 = 40m/s$ $z_u = 25km$ $\Delta_u = 10km$	WKB Euler: $L_z = 40km$ $m \in [-0.01, 0.008]$ WKB Lagrange: $L_z = 40km$ LES: $L_z = 40km, L_x = 10km$	WKB Euler: $nz = 400, nm = 180$ $dz \approx 100m, dm = 10^{-4}s^{-1}$ WKB Lagrange: $nz = 400, dz_{smooth} \approx 600m$ $dz \approx 100m, n_{ray} = 4000$ LES: $nz = 2500, nx = 64$ $dz \approx 16m, dx = 156m$
PREFL Partial Reflection from a jet	Cosine shape $\lambda_x = 6km, \lambda_z = 3km$ $k = 2\pi/\lambda_x, m = 2\pi/\lambda_z$ $z_0 = 10km, \Delta_{wp} = 10km$ $branch = -1, a_0 = 0.1$	non-Boussinesq $T = 300K$ $N \approx 0.018$ $u_0 = 9,75m/s$ $z_u = 25km$ $\Delta_u = 10km$	WKB Lagrange: $L_z = 50km$ LES: $L_z = 50km, L_x = 6km$	WKB Lagrange: $nz = 166, dz_{smooth} \approx 1800m$ $dz \approx 300m, n_{ray} = 4320$ LES: $nz = 538, nx = 32$ $dz \approx 93m, dx = 187m$

STIH Static Instability Hydrostatic Wavepacket	Gaussian shape $\lambda_x = 30km,\ \lambda_z = 3km$ $k = 2\pi/\lambda_x, m = 2\pi/\lambda_z$ $z_0 = 10km, \Delta_{wp} = 25km$ $branch = -1, a_0 = 0.5$	non-Boussinesq $T = 300K$ $N \approx 0.018$	WKB Lagrange: $L_z = 80km$ LES: $L_z = 80km, L_x = 30km$	WKB Lagrange: $nz = 266, dz_{smooth} \approx 1800m$ $dz \approx 300m, n_{ray} = 4320$ LES: $nz = 854, nx = 32$ $dz \approx 94m, dx \approx 940m$
STINH Static Instability Non-hydrostatic Wavepacket	Gaussian shape $\lambda_x = 1km,\ \lambda_z = 1km$ $k = 2\pi/\lambda_x, m = 2\pi/\lambda_z$ $z_0 = 10km, \Delta_{wp} = 10km$ $branch = -1, a_0 = 0.9$	non-Boussinesq $T = 300K$ $N \approx 0.018$	WKB Lagrange: $L_z = 30km$ LES: $L_z = 30km, L_x = 1km$	WKB Lagrange: $nz = 300, dz_{smooth} \approx 600m$ $dz \approx 100m, n_{ray} = 4000$ LES: $nz = 960, nx = 32$ $dz \approx 31m, dx \approx 310m$
MI Modulational Instability	Cosine shape $\lambda_x = 1km,\ \lambda_z = 1km$ $k = 2\pi/\lambda_x, m = 2\pi/\lambda_z$ $z_0 = 10km, \Delta_{wp} = 20km$ $branch = -1, a_0 = 0.1$	non-Boussinesq $T = 300K$ $N \approx 0.018$	WKB Lagrange: $L_z = 60km$ LES: $L_z = 60km, L_x = 1km$	WKB Lagrange: $nz = 600, dz_{smooth} \approx 600m$ $dz \approx 100m, n_{ray} = 4000$ LES: $nz = 1920, nx = 32$ $dz \approx 31m, dx \approx 310m$
CL Critical Layer	Cosine shape $\lambda_x = 10km,\ \lambda_z = 1km$ $k = 2\pi/\lambda_x, m = 2\pi/\lambda_z$ $z_0 = 10km, \Delta_{wp} = 10km$ $branch = -1, a_0 = 0.1$	non-Boussinesq $T = 300K$ $N \approx 0.018$ $u_0 = -11m/s$ $z_u = 25km$ $\Delta_u = 10km$	WKB Lagrange: $L_z = 30km$ LES: $L_z = 30km, L_x = 10km$	WKB Lagrange: $nz = 300, dz_{smooth} \approx 600m$ $dz \approx 100, n_{ray} = 4000$ LES: $nz = 960, nx = 32$ $dz \approx 31m, dx \approx 310m$