

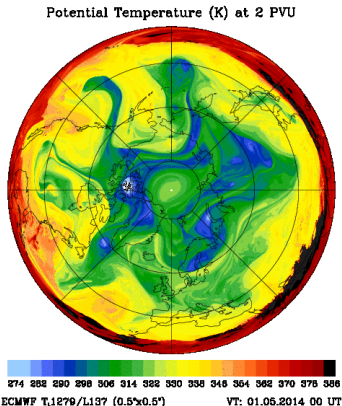
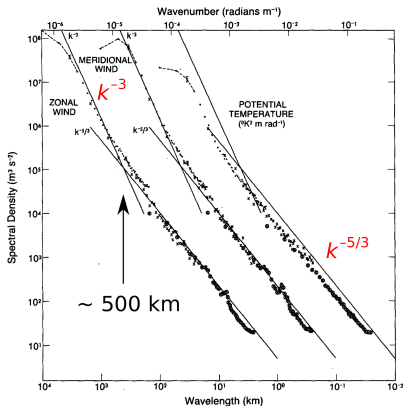
# Wave–vortex decomposition of aircraft data: interpreting the Nastrom–Gage spectrum

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# Nastrom–Gage: the synoptic-to-mesoscale transition



Geostrophic turbulence governs synoptic scales (enstrophy range), but what causes the transition in spectral slope at 500 km?

# The synoptic-to-mesoscale transition: contesting theories

## 1. Inverse energy cascade originating from small scales

*Energy injected by convective activity at small scales cascades to large scales in a quasi-two-dimensional cascade (Gage 1979, Lilly 1983).*

## 2. Frontogenetic processes at the tropopause

*Surface quasi-geostrophic dynamics cause spectral transition (Tulloch and Smith 2006).*

## 3. Ageostrophic stratified turbulence

*Mesoscale flow escapes rotational constraint; energy is cascaded down to small scales (Lindborg 2006).*

## 4. Inertia-gravity waves

*Quasi-linear inertia-gravity waves dominate the mesoscale range (Dewan 1979, VanZandt 1982).*

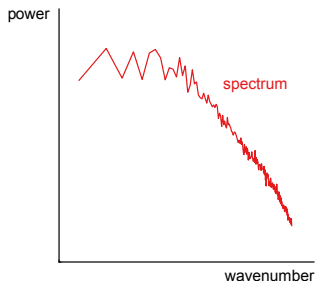
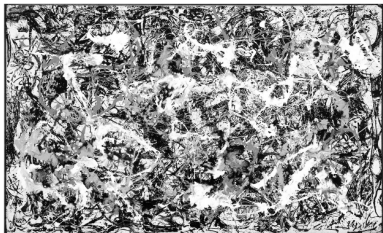
# A theory of Jackson Pollock's art



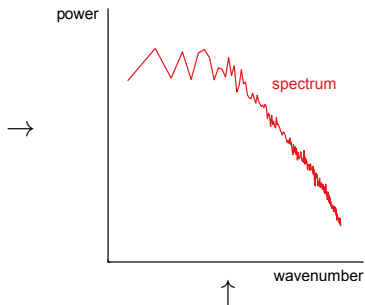
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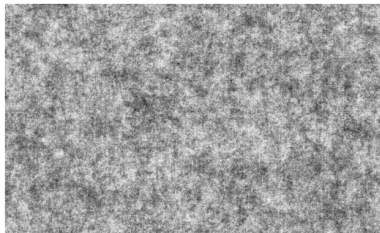
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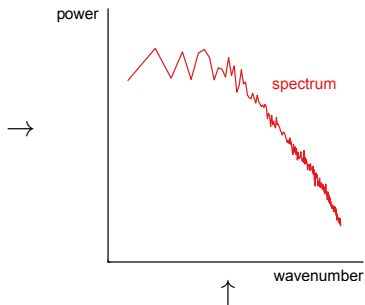


Different theories predict  
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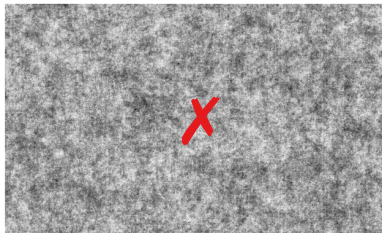
(randomized phase)

# A theory of Jackson Pollock's art



Different theories predict  
**identical spectra**.

Additional information in **the data** must be used to distinguish between theories.



(randomized phase)

*cf. Armi and Flament (1985)*



# Helmholtz decomposition of MOZAIC data

$$S^u(k), S^v(k) \rightarrow K(k) = K^\psi(k) + K^\phi(k)$$



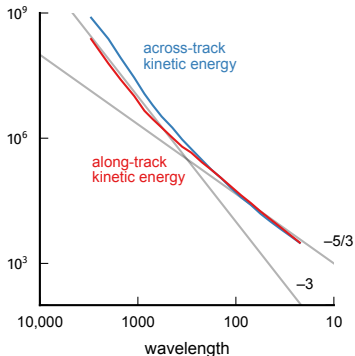
# Helmholtz decomposition of MOZAIC data



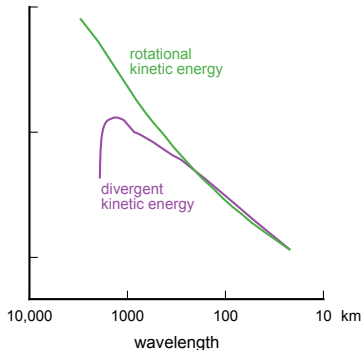
$$S^u(k), S^v(k) \rightarrow K(k) = K^\psi(k) + K^\phi(k)$$

power spectral density ( $\text{m}^3\text{s}^{-2}$ )

observed spectra



Helmholtz decomposition



The **synoptic** scales are dominated by the **rotational** component.

The **mesoscales** have a significant **divergent** component.

# The synoptic-to-mesoscale transition: contesting theories

## 1. Inverse energy cascade originating from small scales **X**

*Energy injected by convective activity at small scales cascades to large scales in a quasi-two-dimensional cascade (Gage 1979, Lilly 1983).*

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## Do linear waves dominate at mesoscales?

Check whether observations are consistent with **polarization** and **dispersion relations** of inertia–gravity waves.

Energy partition statement from **linear wave theory**:

$$E_w(k) = 2K^\phi(k) \quad (\text{hydrostatic})$$

$$E_w(k) = 2K^\phi(k) + S^w(k) \quad (\text{nonhydrostatic})$$

Does the diagnosed  $E_w(k)$  match the observed  $E(k)$ ?

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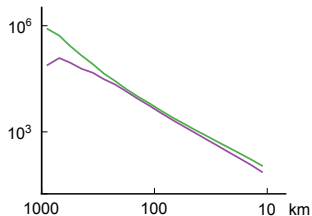
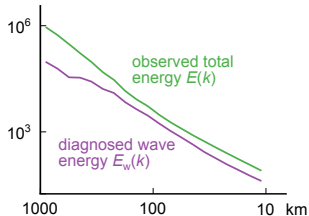
Vertical homogeneity assumption: density-weighted spectra, phase averaging by variable flight altitude

# Wave-vortex decomposition

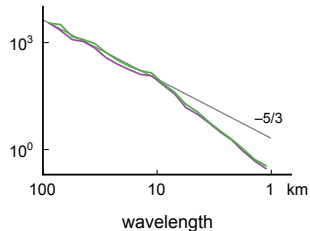
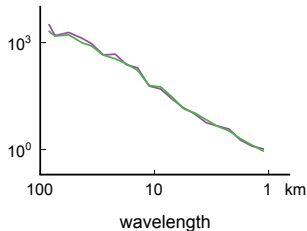
power spectral density ( $\text{kg s}^{-2}$ )

upper troposphere

lower stratosphere

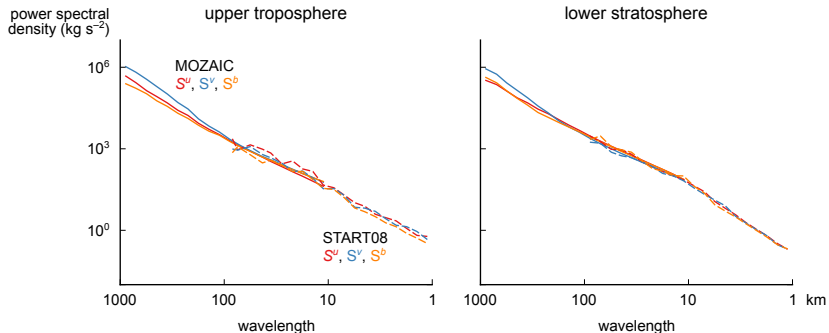


GV (START08)



*cf. Zhang et al. (2015)*

# Source of inconsistency between the data sets



The two data sets disagree in the **along-track velocity** component at wavelengths 10–100 km in the **upper troposphere**.

# Conclusions

Mesoscale flows have significant **divergence**, which excludes theories based on quasi-geostrophic dynamics.

The mesoscale spectra are consistent with **linear inertia-gravity waves** in both upper troposphere and lower stratosphere.

There is a mismatch in the MOZAIC upper-tropospheric diagnosis, where there is an **inconsistency** between data sets.

Stratified turbulence cannot explain the **steepening** of stratospheric spectra at wavelengths smaller than 10 km.