Superpressure Balloon Studies of Atmospheric Gravity Waves in the Stratosphere

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Introduction

- Super pressure balloons are a powerful way to derive gravity wave momentum fluxes
- Problems occur at periods shorter than about 20 min
- High precision GPS measurements offer a way to derive momentum fluxes at short periods

SPB Equation of Motion

Following Nastrom (1980) the equation of motion in the vertical direction is:

$$(M_B + \eta M_a)\frac{\partial^2 \zeta_b'}{\partial t^2} = -g(M_B - M_a) - \frac{1}{2}\rho_a C_d A_B \left(\frac{\partial \zeta_b'}{\partial t} - w'\right)\left|\frac{\partial \zeta_b'}{\partial t} - w'\right| + (M_B + \eta M_a)\frac{\partial w'}{\partial t}$$

i.e. net force = buoyancy force + drag force + dynamic force

This can be simplified to

$$\frac{\partial^2 \zeta_b'}{\partial t^2} = -\omega_B^2 \zeta_b' + \frac{2}{3} g R - A\left(\frac{\partial \zeta_b'}{\partial t} - w'\right)\left|\frac{\partial \zeta_b'}{\partial t} - w'\right| + \frac{\partial w'}{\partial t} \qquad (1)$$

where

$$\omega_B^2 = \frac{2g}{3T}\left(\frac{\partial T}{\partial z} + \frac{g}{R_a}\right)$$

$$A = \frac{C_d}{4r}$$

R = wave induced density perturbation

ω_B = Buoyancy frequency

For a gravity wave of intrinsic frequency $\hat{\omega}$ and vertical velocity amplitude w_o the instantaneous vertical velocity is $w' = w_o e^{-i\hat{\omega}t}$

$$R = \frac{\rho'}{\overline{\rho}} = i\frac{N^2}{g\hat{\omega}}w'$$

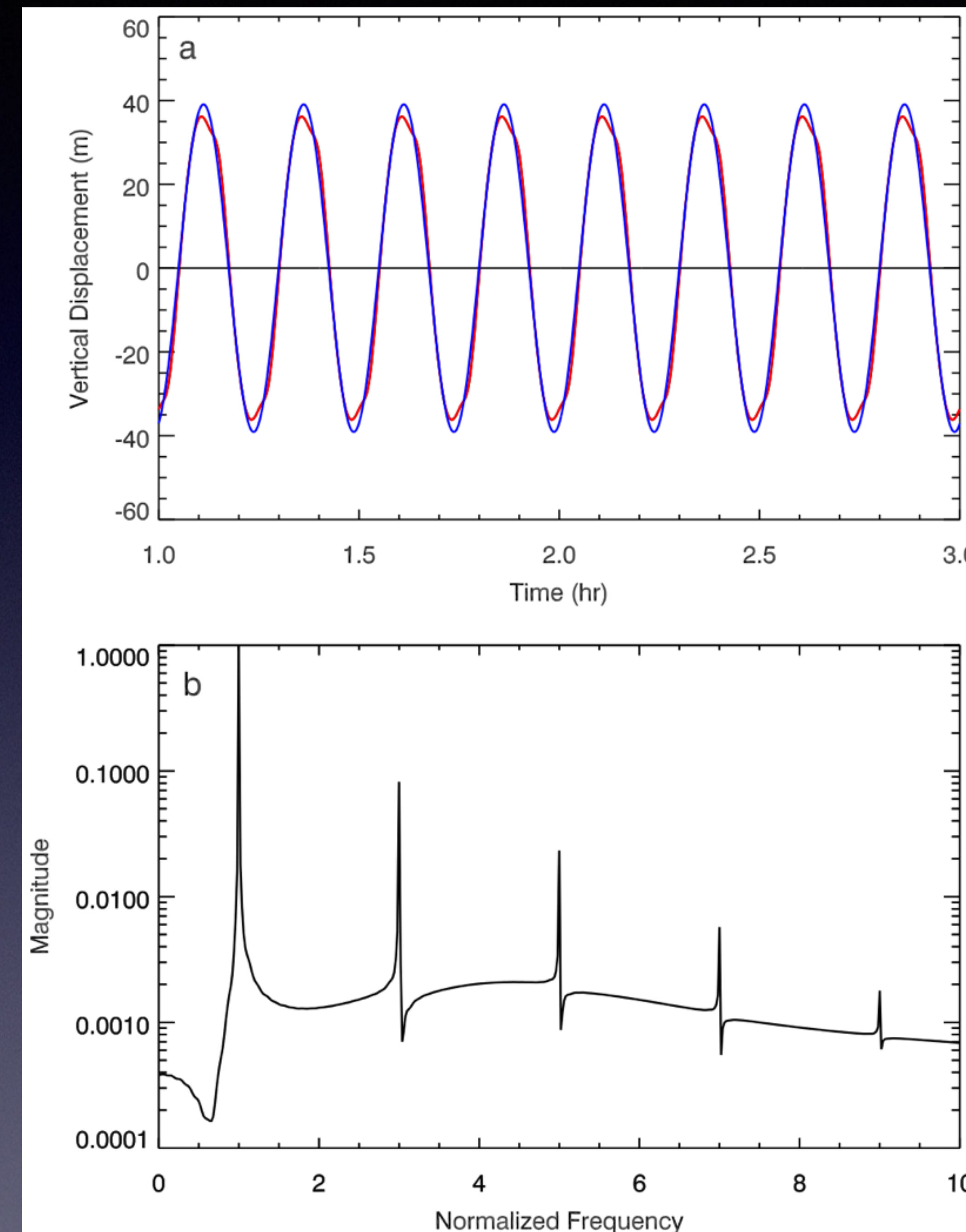
$$N^2 = \frac{g}{T}\left(\frac{g}{c_p} + \frac{\partial T}{\partial z}\right)$$

(1) can be solved numerically

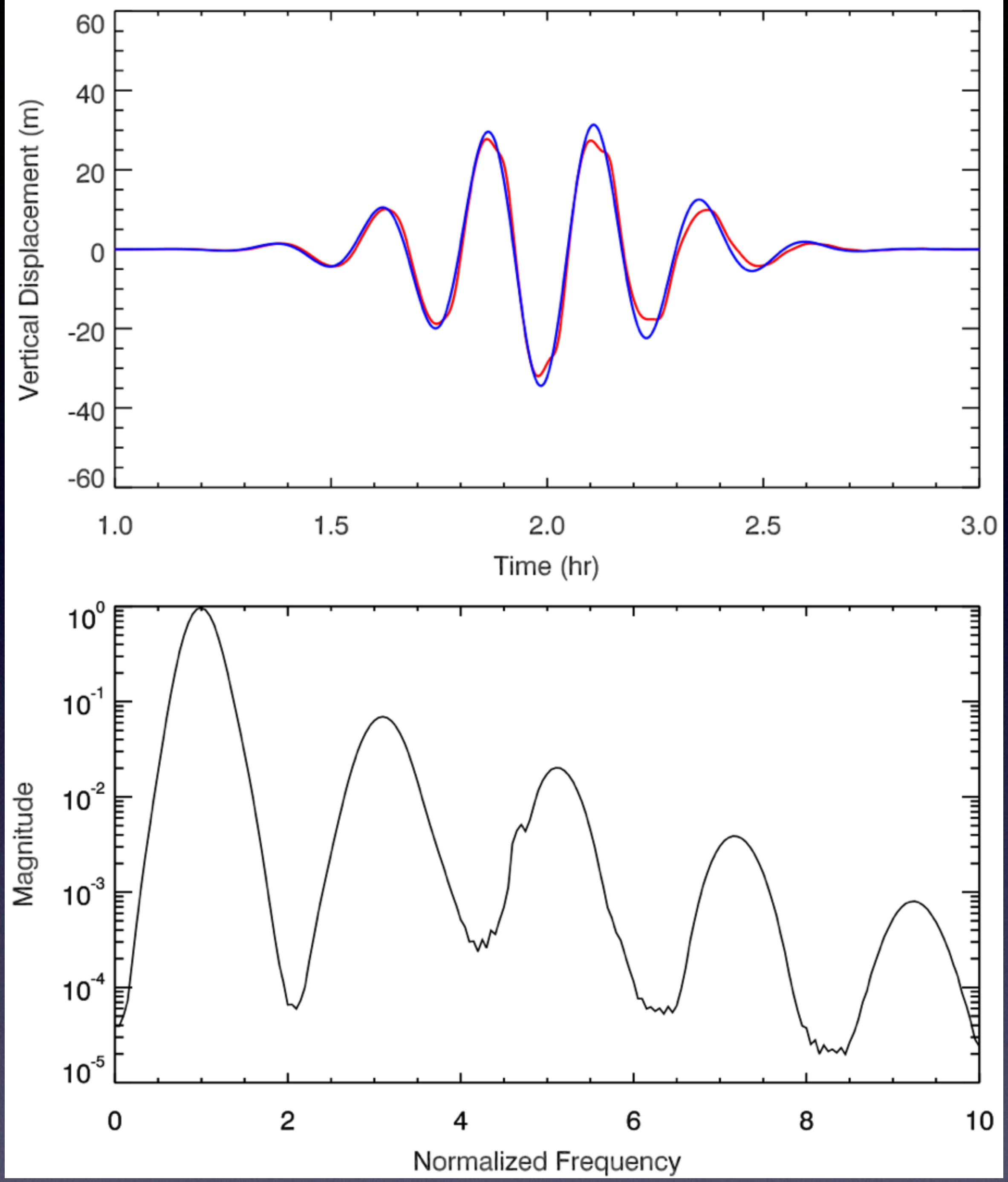
Example:
Wave with 15 min intrinsic period, w_o=1 ms^{-1}

Red curve is numerical solution

Response contains only odd harmonics



Wave packet $\hat{\tau} = 15$ min



Harmonic distortion increases as wave amplitude increases

Quasi-Analytic Solution

If the vertical displacement of air parcel is ζ' then let $\frac{\zeta'_b}{\zeta'} = \underline{Z} = |Z|e^{i\varphi}$

(1) becomes:

$$-\hat{\omega}^2\zeta'_b = -\omega_B^2\zeta'_b + \tfrac{2}{3}N^2\zeta' - A(-i\hat{\omega}\zeta'_b + i\hat{\omega}\zeta')|-i\hat{\omega}\zeta'_b + i\hat{\omega}\zeta'| - \hat{\omega}^2\zeta'$$

Hence:

$$\underline{Z} = \frac{\frac{2}{3}N^2 - \hat{\omega}^2 - iA\hat{\omega}^2\zeta_o Y}{\omega_B^2 - \hat{\omega}^2 - iA\hat{\omega}^2\zeta_o Y} \qquad (2)$$

where $Y = |1 - \underline{Z}|$

Iterative solution for (2) with quick convergence.

Note the limiting values of $|\underline{Z}|$ and φ when the balloon is on its equilibrium density surface (EDS) - *isopycnic balloon*.

From (2) $|\underline{Z}_{EDS}| \to \frac{2N^2}{3\omega_B^2} = \frac{\left(\frac{g}{c_p} + \frac{dT}{dz}\right)}{\left(\frac{g}{R_a} + \frac{dT}{dz}\right)} \approx 0.3$

For reference

Antarctic:

$N = 0.023$ rad s^{-1} $\quad \tau_N = 4.6$ min

$\omega_B = 0.034$ rad s^{-1} $\quad \tau_B = 3.1$ min

$Z_{EDS} = 0.31$

Equator:

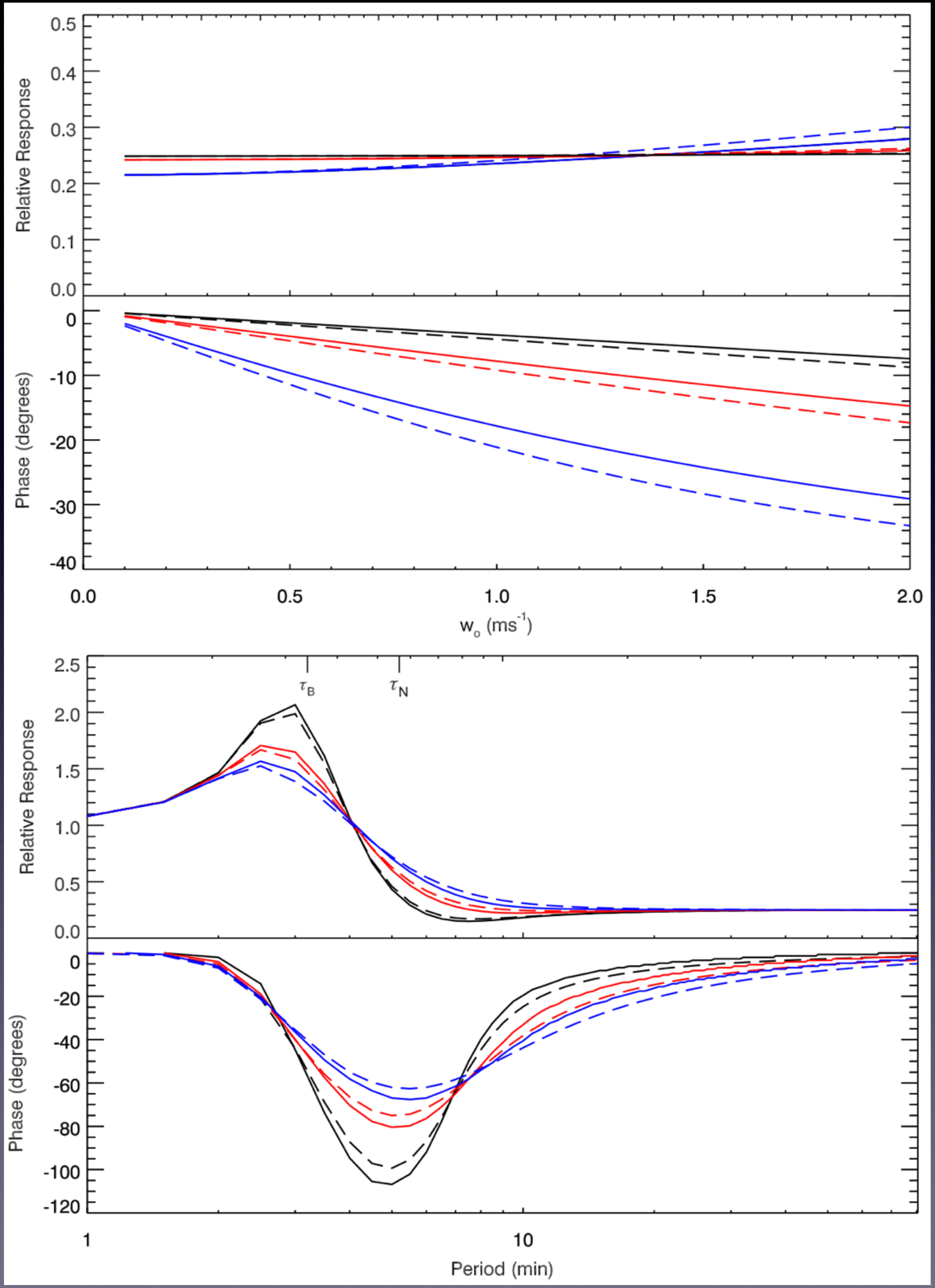
$N = 0.025$ rad s^{-1} $\quad \tau_N = 4.2$ min

$\omega_B = 0.035$ rad s^{-1} $\quad \tau_B = 3.0$ min

$Z_{EDS} = 0.34$

$\left|\frac{\zeta_b}{\zeta}\right|$

φ



$\left|\frac{\zeta_b}{\zeta}\right|$

φ

$\hat{\tau} = 60$ min

$\hat{\tau} = 15$ min

Note significant phase shift, $\varphi(\hat{\omega})$, when $\hat{\tau} \lesssim 20$ min

SPB Measurements

Horizontal position: $x(t), y(t) \rightarrow u(t), v(t)$

Vertical position: ζ_b

Pressure: $p'_T = p' + \frac{d\overline{p}}{dz}\zeta'_b$

$$p' = p'_T + \overline{\rho} g \zeta'_b$$

Temperature: $T'_T = T' + \frac{d\overline{T}}{dz}\zeta'_b$

Process observations in wavelet space as a function of $(\hat{\omega}, t)$

$$\mathrm{Im}\left(\tilde{p}_T \tilde{u}_{||}\right) = -\overline{\rho} H \frac{N^2}{\hat{\omega}} \mathrm{Re}\left(\tilde{u}^*_{||} \tilde{w}\right)$$

Systematic errors in flux retrieval for $\hat{\tau} \lesssim 20$ min

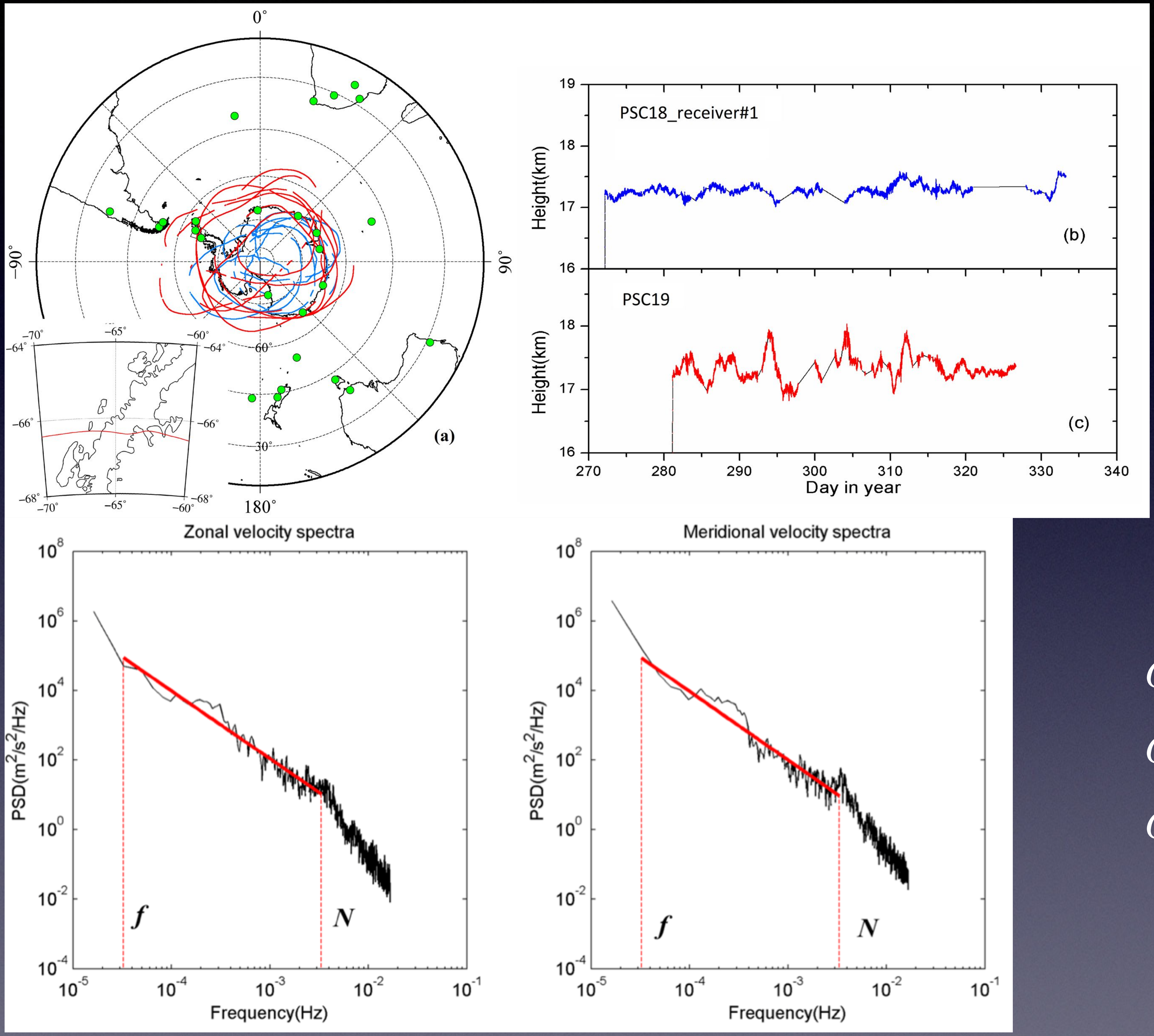
Characteristics of GW Generated by Convection

Significant part of momentum flux spectrum at periods less than 20 min

Typical phase speeds $c \sim 0 - 60\ \mathrm{ms}^{-1}$

Typical horizontal wavelengths $\lambda_h \sim 10 - 100$ km

High-Resolution GPS Observations



$$\sigma_h \sim 0.1 \text{ m}$$
$$\sigma_u \sim 0.003 \text{ ms}^{-1}$$
$$\sigma_{\zeta_b} \sim 0.2 \text{ m}$$

Zhang et al., (2016) Improvement of stratospheric balloon GPS positioning and the impact on gravity wave parameter estimation for the Concordiasi campaign in Antarctica, JGR, (under review).

I - Direct Computation of w′

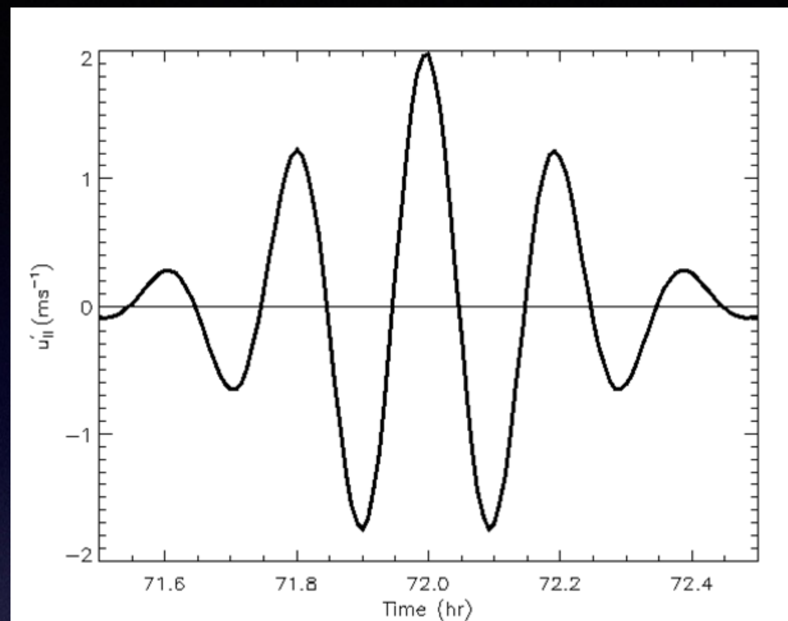
(Simple version)

1. In wavelet space find peak value of $\tilde{u}_{||}^2 \rightarrow t_{max}, \tilde{\omega}_{max}$
2. Estimate peak value of ζ_b at this location
3. Find range of periods that are significant
4. Make an initial guess of $\zeta_o = \zeta_b / Z_{EDS}$
5. Compute $\underline{Z}$ from (2), derive new guess of $\zeta_o = \zeta_b / Re(\underline{Z})$
6. Iterate until convergence $\rightarrow \underline{Z}_f \rightarrow \zeta(\tilde{\omega}, t) = \zeta_b(\tilde{\omega}, t) / \underline{Z}_f$
7. Compute fluxes directly in wavelet space

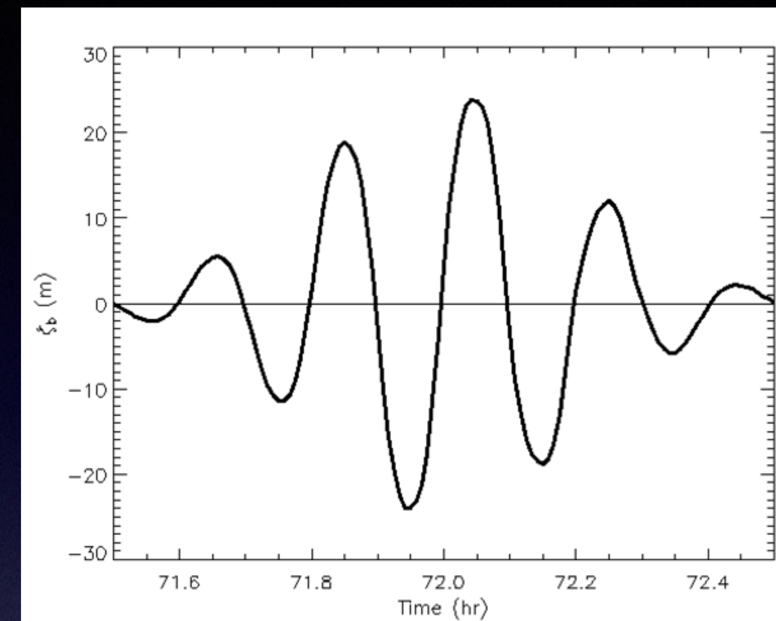
II -Direct Computation of w′

Input

$u'_{||}$



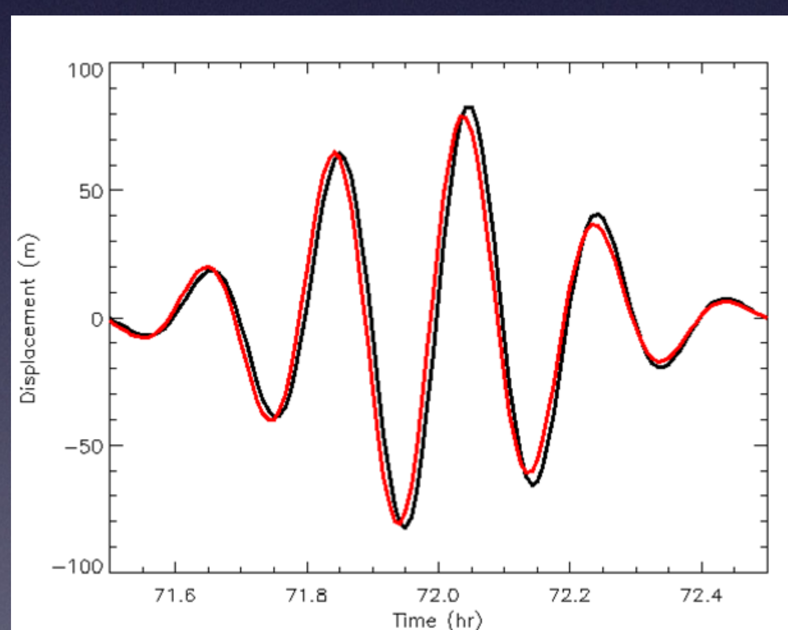
ζ'_b



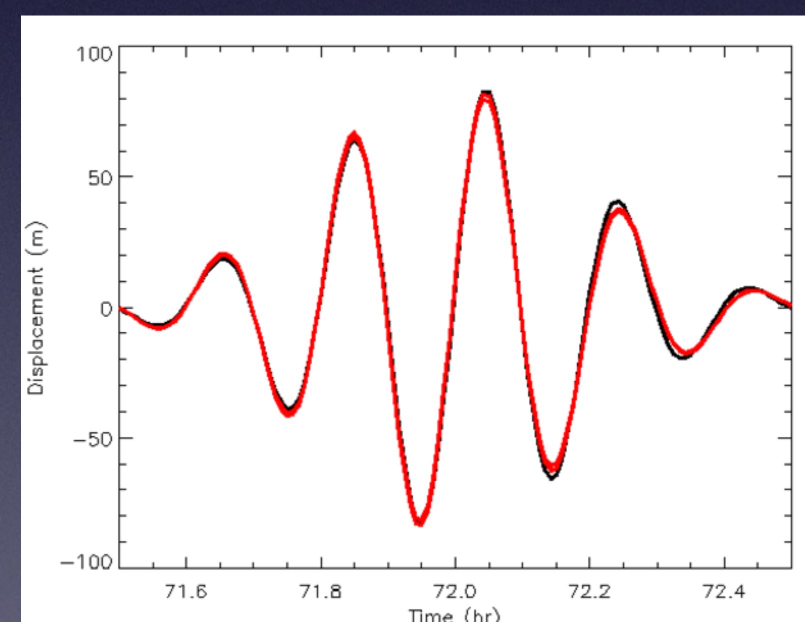
Notice distortion

$$\hat{\tau} = 12 \text{ min}$$
$$\hat{c} = 25 \text{ ms}^{-1}$$
$$\theta = 45°$$
$$u_{||} = 2 \text{ ms}^{-1}$$

Output

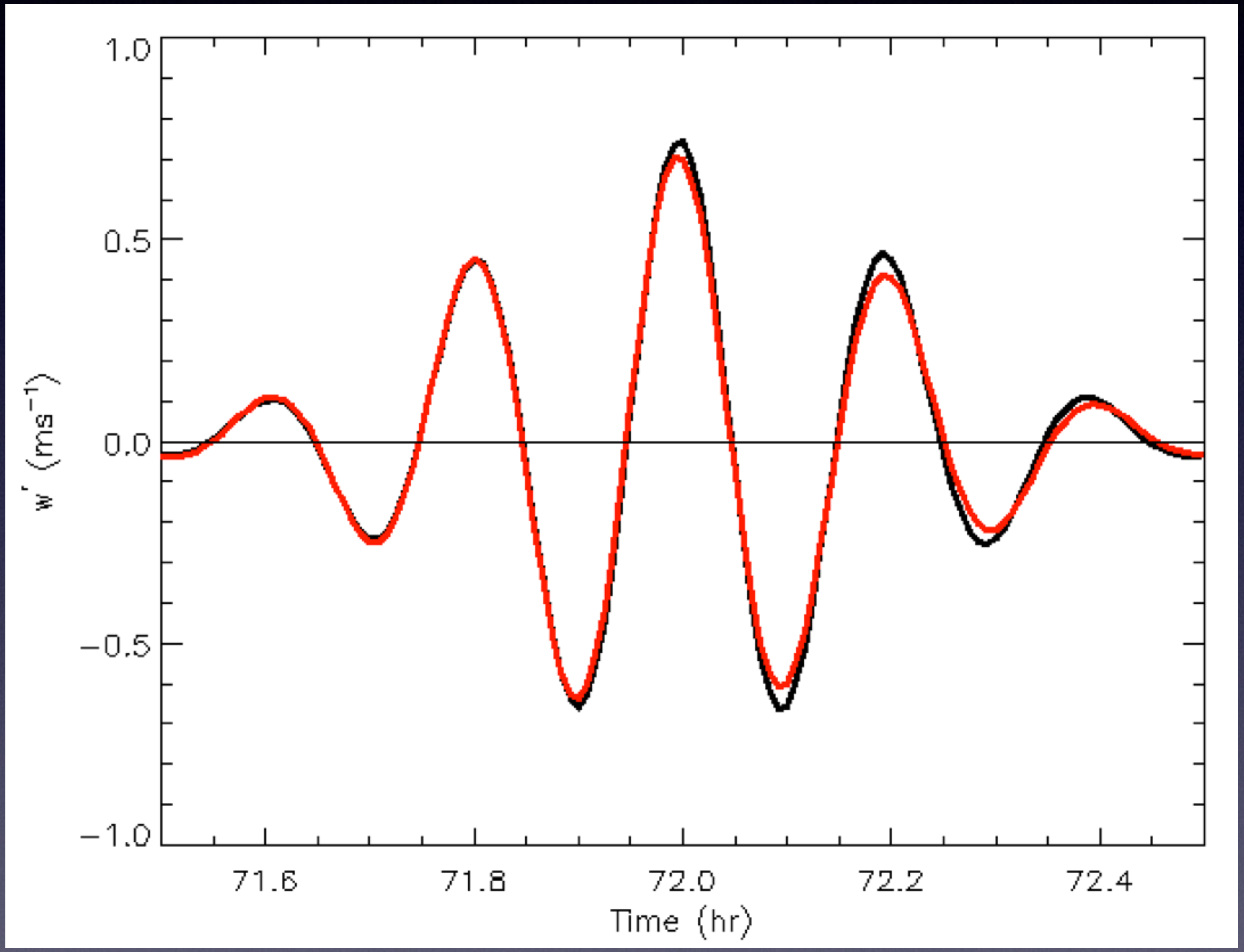


ζ' before phase correction



ζ' after phase correction

III-Direct Computation of w′



Flux Retrievals - Single Wave

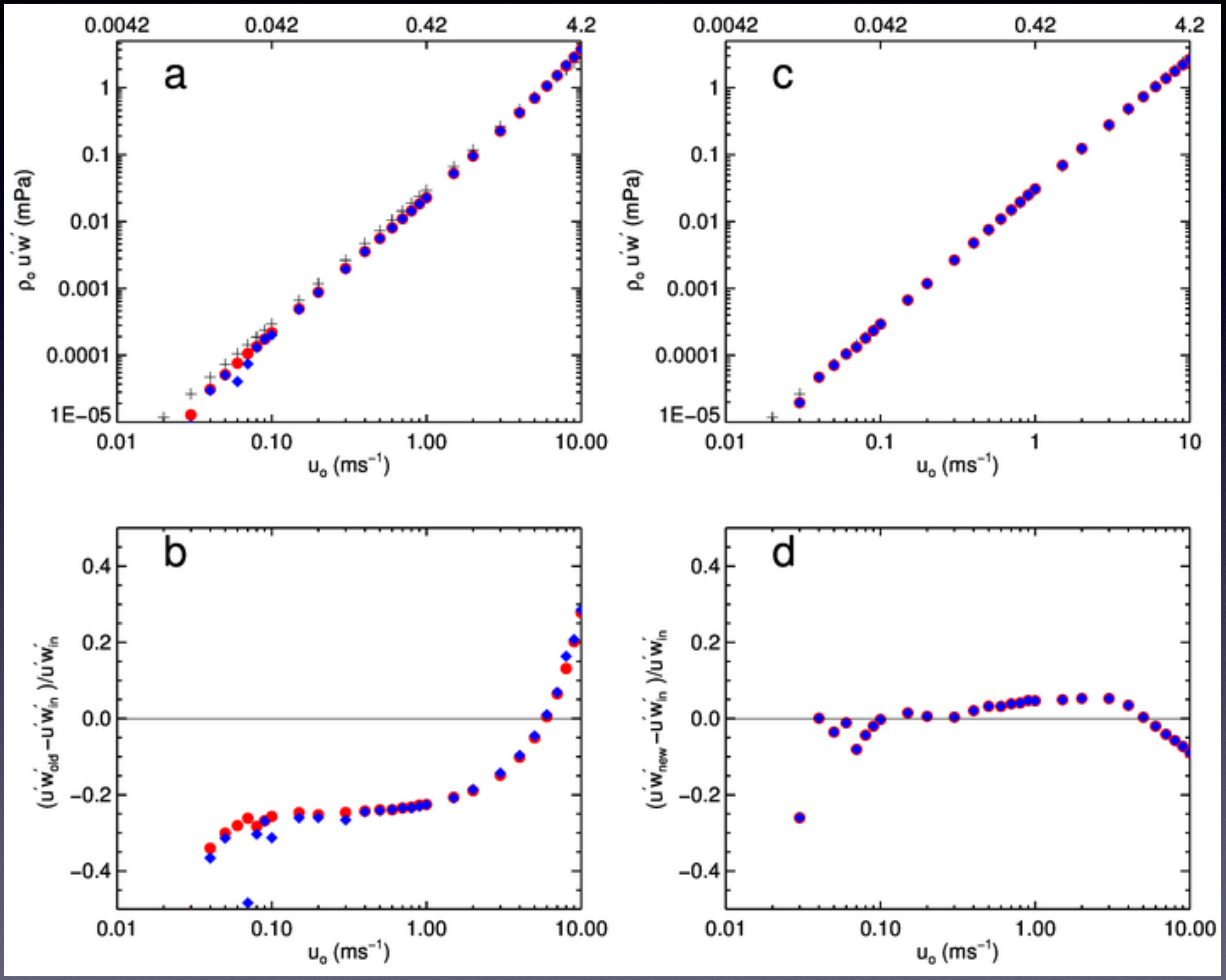
$\hat{\tau} = 10$ min

$\hat{c} = 26\ \mathrm{ms}^{-1}$

$\theta = 135°$

With noise

Without noise



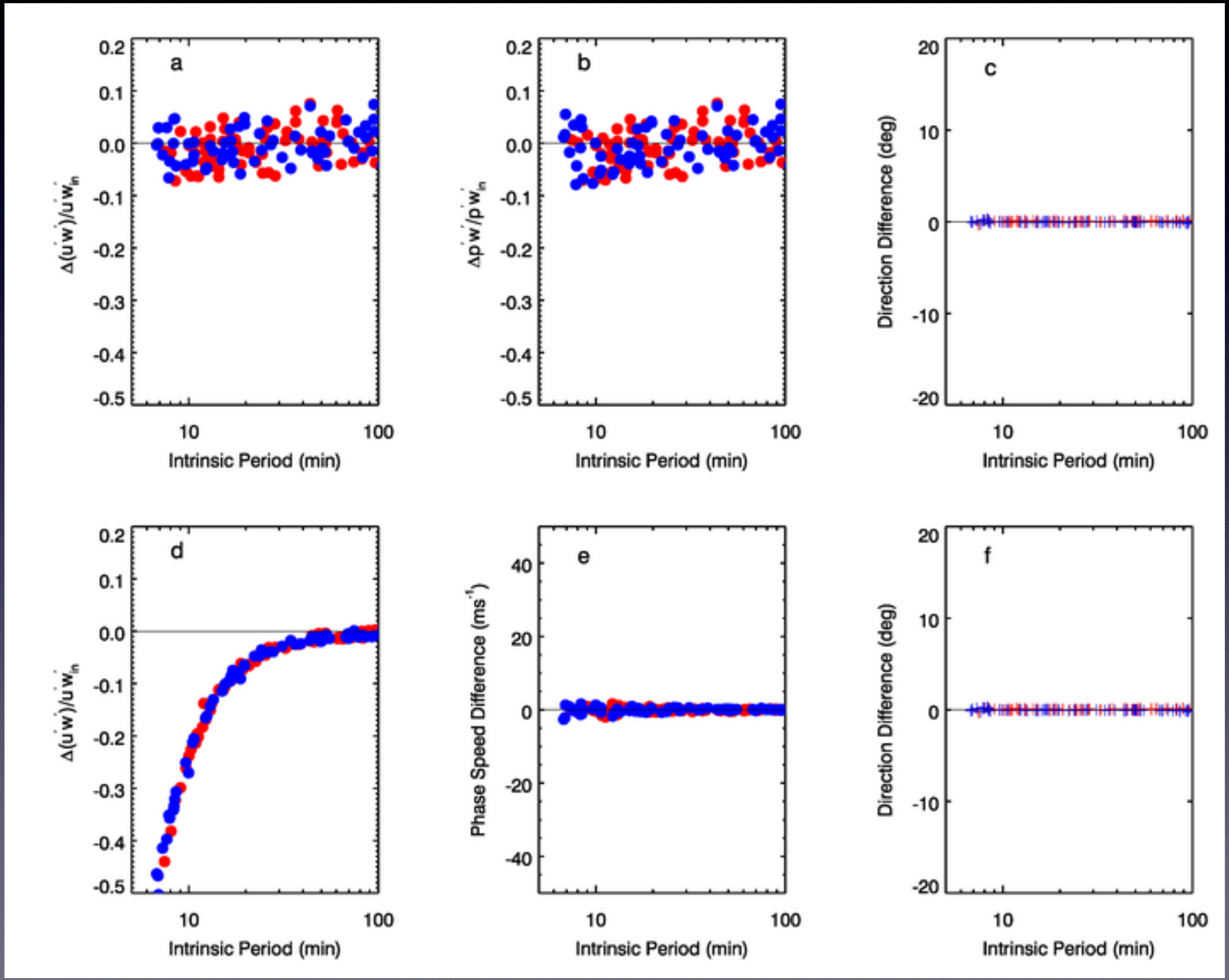
Old Method

New Method

Flux Retrievals - Multiple Waves

Down going

Up going



Conclusions

- Objective method for converting vertical SPB displacements to vertical parcel displacements
- Requires high resolution GPS measurements of SPB position in horizontal and vertical
- Estimate fractional wave fluxes to ~2%
- Results important for flux retrievals over convection
- Easy to extend to derive temperature perturbations