Superpressure Balloon Studies of Atmospheric Gravity Waves in the Stratosphere

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Introduction

- Super pressure balloons are a powerful way to derive gravity wave momentum fluxes
- Problems occur at periods shorter than about 20 min
- High precision GPS measurements offer a way to derive momentum fluxes at short periods
Following Nastrom (1980) the equation of motion in the vertical direction is:

\[(M_B + \eta M_a) \frac{\partial^2 \zeta_b'}{\partial t^2} = -g(M_B - M_a) - \frac{1}{2} \rho_a C_d A_B \left( \frac{\partial \zeta_b'}{\partial t} - w' \right) \left| \frac{\partial \zeta_b'}{\partial t} - w' \right| + \]

\[(M_B + \eta M_a) \frac{\partial w'}{\partial t} \]

i.e. net force = buoyancy force + drag force + dynamic force

This can be simplified to

\[\frac{\partial^2 \zeta_b'}{\partial t^2} = -\omega_B^2 \zeta_b' + \frac{2}{3} gR - A \left( \frac{\partial \zeta_b'}{\partial t} - w' \right) \left| \frac{\partial \zeta_b'}{\partial t} - w' \right| + \frac{\partial w'}{\partial t} \quad (1)\]

where

\[\omega_B^2 = \frac{2g}{3T} \left( \frac{\partial T}{\partial z} + \frac{g}{R_a} \right)\]

\[A = \frac{C_d}{4r}\]

\[R = \text{wave induced density perturbation}\]

\[\omega_B = \text{Buoyancy frequency}\]
For a gravity wave of intrinsic frequency $\hat{\omega}$ and vertical velocity amplitude $w_o$ the instantaneous vertical velocity is $w' = w_o e^{-i\hat{\omega}t}$

$$R = \frac{\rho'}{\rho} = i \frac{N^2}{g\hat{\omega}} w'$$

$$N^2 = \frac{g}{T} \left( \frac{g}{c_p} + \frac{\partial T}{\partial z} \right)$$

(1) can be solved numerically

Example:
Wave with 15 min intrinsic period, $w_o = 1 \text{ ms}^{-1}$

Red curve is numerical solution

Response contains only odd harmonics
Wave packet $\hat{\tau} = 15$ min

Harmonic distortion increases as wave amplitude increases
If the vertical displacement of air parcel is $\zeta'$ then let $\frac{\zeta_b'}{\zeta'} = Z = |Z|e^{i\varphi}$

(1) becomes:

$$-\hat{\omega}^2 \zeta_b' = -\omega_B^2 \zeta_b' + \frac{2}{3}N^2 \zeta' - A(-i\hat{\omega}\zeta_b' + i\hat{\omega}\zeta')| - i\hat{\omega}\zeta_b' + i\hat{\omega}\zeta'|- \hat{\omega}^2 \zeta'$$

Hence:

$$Z = \frac{\frac{2}{3}N^2 - \hat{\omega}^2 - iA\hat{\omega}^2 \zeta_o Y}{\omega_B^2 - \hat{\omega}^2 - iA\hat{\omega}^2 \zeta_o Y}$$

(2)

where $Y = |1 - Z|$ 

Iterative solution for (2) with quick convergence.
Note the limiting values of $|Z|$ and $\varphi$ when the balloon is on its equilibrium density surface (EDS) - *isopycnic balloon.*

From (2) $|Z_{EDS}| \rightarrow \frac{2N^2}{3\omega_B^2} = \left( \frac{g}{c_p} + \frac{dT}{dz} \right) \left( \frac{g}{R_a} + \frac{dT}{dz} \right) \approx 0.3$

For reference
Antarctic:
- $N = 0.023 \text{ rad s}^{-1}$
- $\omega_B = 0.034 \text{ rad s}^{-1}$
- $Z_{EDS} = 0.31$
- $\tau_N = 4.6 \text{ min}$
- $\tau_B = 3.1 \text{ min}$
Equator:
- $N = 0.025 \text{ rad s}^{-1}$
- $\omega_B = 0.035 \text{ rad s}^{-1}$
- $Z_{EDS} = 0.34$
- $\tau_N = 4.2 \text{ min}$
- $\tau_B = 3.0 \text{ min}$
Note significant phase shift, $\varphi(\hat{\omega})$, when $\hat{\tau} \lesssim 20$ min

$\hat{\tau} = 60$ min

$\hat{\tau} = 15$ min
Horizontal position: $x(t), y(t) \rightarrow u(t), v(t)$

Vertical position: $\zeta_b$

Pressure:

\[
p_T' = p' + \frac{d\bar{p}}{dz} \zeta'_b
\]

\[
p' = p_T' + \bar{\rho}g \zeta'_b
\]

Temperature:

\[
T_T' = T' + \frac{dT}{dz} \zeta'_b
\]

Process observations in wavelet space as a function of $(\hat{\omega}, t)$

\[
\text{Im}(\tilde{p}_T \tilde{u}_{\|}) = -\bar{\rho}H\frac{N^2}{\hat{\omega}} \text{Re} \left( \tilde{u}_{\|}^* \tilde{w} \right)
\]

Systematic errors in flux retrieval for $\hat{\tau} \lesssim 20 \text{ min}$
Characteristics of GW Generated by Convection

Significant part of momentum flux spectrum at periods less than 20 min

Typical phase speeds \( c \sim 0 - 60 \text{ ms}^{-1} \)

Typical horizontal wavelengths \( \lambda_h \sim 10 - 100 \text{ km} \)
High-Resolution GPS Observations

Zhang et al., (2016) Improvement of stratospheric balloon GPS positioning and the impact on gravity wave parameter estimation for the Concordiasi campaign in Antarctica, JGR, (under review).

\[ \sigma_h \sim 0.1 \text{ m} \]
\[ \sigma_u \sim 0.003 \text{ ms}^{-1} \]
\[ \sigma_{\zeta_b} \sim 0.2 \text{ m} \]
I - Direct Computation of \( w' \)
(Simple version)

1. In wavelet space find peak value of \( \tilde{u}^2 \rightarrow t_{max}, \tilde{\omega}_{max} \)
2. Estimate peak value of \( \zeta_b \) at this location
3. Find range of periods that are significant
4. Make an initial guess of \( \varsigma_o = \frac{\zeta_b}{Z_{EDS}} \)
5. Compute \( Z \) from (2), derive new guess of \( \varsigma_o = \frac{\zeta_b}{Re(Z)} \)
6. Iterate until convergence \( \rightarrow Z_f \rightarrow \zeta(\tilde{\omega}, t) = \frac{\zeta_b(\tilde{\omega}, t)}{Z_f} \)
7. Compute fluxes directly in wavelet space
II - Direct Computation of $w'$

**Input**

- $u'_\parallel$
- $\zeta'_b$

**Output**

- $\zeta'$ before phase correction
- $\zeta'$ after phase correction

Notice distortion

- $\hat{\tau} = 12 \text{ min}$
- $\hat{c} = 25 \text{ ms}^{-1}$
- $\theta = 45^\circ$
- $u'_\parallel = 2 \text{ ms}^{-1}$
III-Direct Computation of $w'$
Flux Retrievals - Single Wave

\( \hat{\tau} = 10 \text{ min} \)

\( \hat{c} = 26 \text{ ms}^{-1} \)

\( \theta = 135^\circ \)

<table>
<thead>
<tr>
<th>With noise</th>
<th>Without noise</th>
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<tbody>
<tr>
<td>Old Method</td>
<td>New Method</td>
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\[ \rho_u u' (\text{mPa}) \]

\[ (u_{w_{\text{obs}}} - u_{w_{\text{mod}}}) / u_{w_{\text{obs}}} \]

\[ u_0 (\text{ms}^{-1}) \]
Flux Retrievals - Multiple Waves

Down going
Up going
Conclusions

- Objective method for converting vertical SPB displacements to vertical parcel displacements
- Requires high resolution GPS measurements of SPB position in horizontal and vertical
- Estimate fractional wave fluxes to ~2%
- Results important for flux retrievals over convection
- Easy to extend to derive temperature perturbations