Gravity wave emission and propagation in the differentially heated rotating annulus experiment

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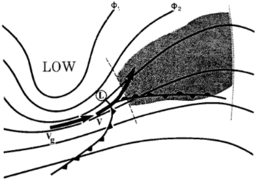
Introduction and Motivation

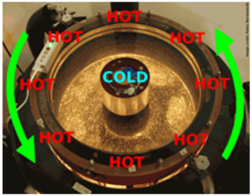
- Orographic and convective sources of inertia-gravity waves (IGW) well understood (Fritts (2003))
- Spontaneously emitted GWs active field of research (Cámara and Lott (2015), ...)
- Observation identify increased IGW activity in jet exit regions (e.g. Plougonven and Zhang (2014))
- GW emission embedded in various atmospheric processes
- Need for controllable, repeatable and simplified laboratory experiments: **rotating annulus**



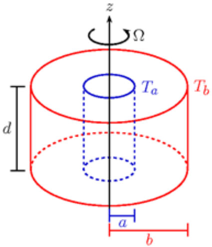
Koch and OHandley (1997)

Introduction and Motivation

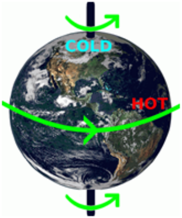
- Differentially heated rotating annulus experiment:



Laboratory experiment (BTU Cottbus)



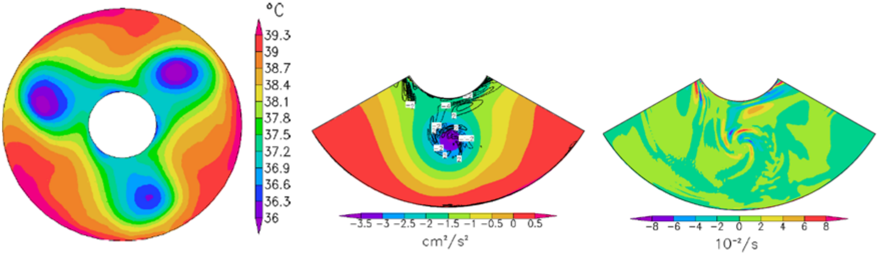
schematic view



analogous to Earth's atmosphere

- Finite volume code (cylFloit) to simulate experiment using Boussinesq approximation (Borchert et al. (2015))

- Annulus simulations show clear GW activity (Borchert et al. (2014))

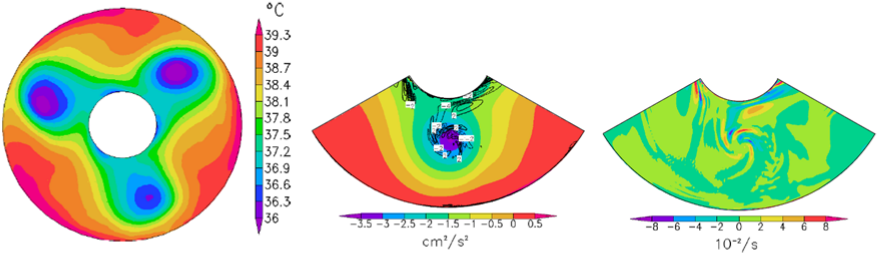


Development of baroclinic waves

Pressure and horizontal divergence

$\delta = \nabla_h \cdot \vec{u}$

- Annulus simulations show clear GW activity (Borchert et al. (2014))



Development of baroclinic waves   Pressure and horizontal divergence $\delta = \nabla_h \cdot \vec{u}$

- Further understanding of GW source processes: tangent linear analysis (see Snyder et al. (2009), Wang and Zhang (2010))

$\Rightarrow$ **Is there some internal forcing of GWs by the balanced flow?**

Source mechanism of gravity wave emission

Decomposition of flow into geostrophic and ageostrophic part

$$
\begin{aligned}
\boldsymbol{v} &= \boldsymbol{u_g} + \boldsymbol{v_a} \\
B &= B_g + B_a \\
p &= p_g + p_a
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\begin{aligned}
f\boldsymbol{e_z} \times \boldsymbol{u_g} + \nabla_h p_g &= 0 \\
B_g - \frac{\partial p_g}{\partial z} &= 0 \\
\Pi_g = \zeta + \frac{f}{N^2}\frac{\partial B}{\partial z} &= \frac{1}{f}\left(\nabla_h^2 + \frac{f^2}{N^2}\frac{\partial^2}{\partial z^2}\right) p_g \\
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... $\Rightarrow$ Geostrophic forcing of ageostrophic flow

$$
\frac{D\delta_a}{Dt} = -\frac{\partial B_a}{\partial z} + \frac{\partial^2 p_{aa}}{\partial z^2} + \frac{\partial \boldsymbol{v}}{\partial z} \cdot w_a - \boxed{\frac{\partial^2}{\partial z^2}\nabla^{-2}\left(\nabla \boldsymbol{u_g} \cdot\cdot \nabla \boldsymbol{u_g}\right)}
$$

Tangent Linear Analysis

Linearisation of unbalanced flow about balanced flow: Principle

- Decomposition into balanced (large) and unbalanced (small) part

  $x = \tilde{x} + x'$ with $|x'| << |\tilde{x}|$ (**Unbalanced part≡gravity waves**)

- Tangent linear evolution of x'

$$\frac{\partial x'}{\partial t} = L(\tilde{x})x' + F(\tilde{x})$$

  with a linear operator $L(\tilde{x})$ and a balanced forcing term $F(\tilde{x})$
- Unbalanced component is integrated separately within each time step
- Balanced part serves as background of the tangent linear model

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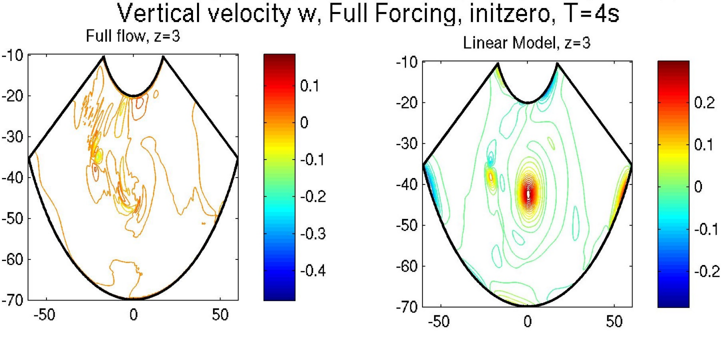
- **Balanced forcing leading contributor to the gravity wave activity?**

Results

Problem: Instabilities at side walls

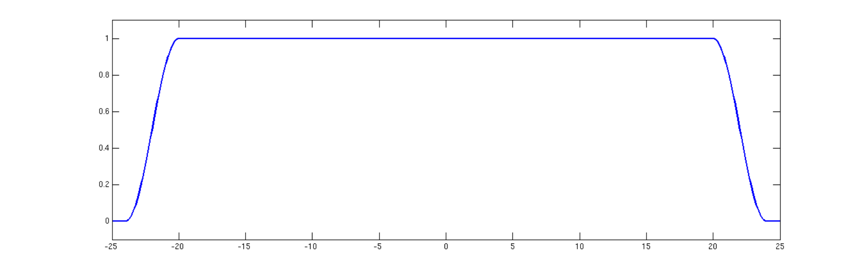
- linear model diverges after about 4-5 s of integration time
- Exponential growth rate at outer (and inner) side walls



Results

Suppress growth at side wall: Multiplication with window function

$$f(x) = \begin{cases} 1, & |x| \leq \beta L_y \\ \frac{1}{2}\left\{1 + cos\left[\frac{\pi(|x|-\beta L_y)}{L_s}\right]\right\}, & \beta L_y < |x| \leq [\beta + \gamma(1-\beta)]\, L_y \\ 0, & else \end{cases}$$



Comparison of horizontal divergence

- Initialising linear model with zero unbalanced part
- Full forcing
- T=22 s ($\Omega = 0.08$ rad/s, rotation period ~ 79 s)

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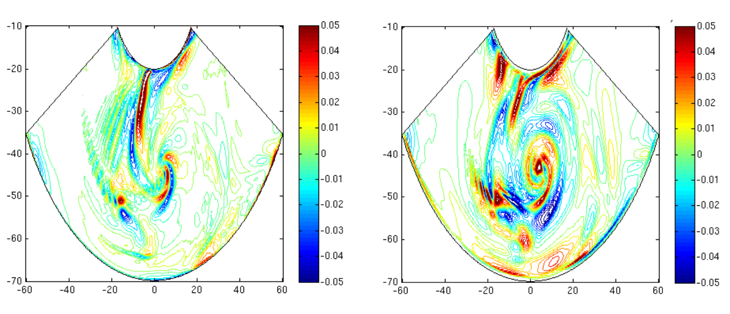

nonlinear model linear model

Results

Comparison of horizontal divergence

- Initialising linear model with $x'_{init} = x - \tilde{x}$
- T=0 s

Results

Comparison of horizontal divergence

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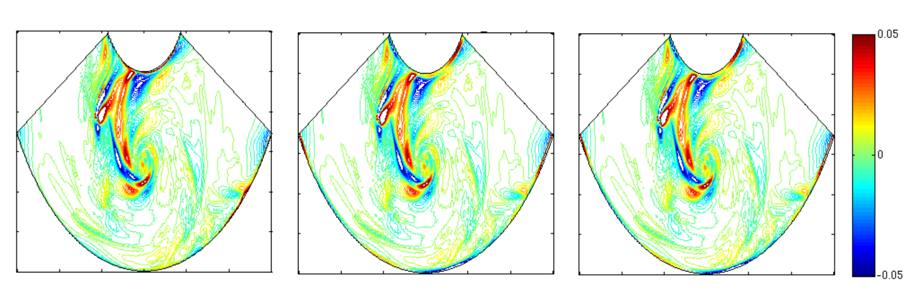
- T=0 s



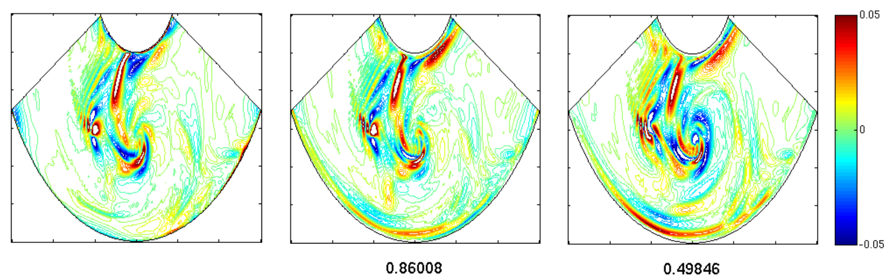
nonlinear model   linear with forcing   linear without forcing

Comparison of horizontal divergence

- Initialising linear model with $x'_{init} = x - \tilde{x}$
- T=10 s

Comparison of horizontal divergence

- Initialising linear model with $x'_{init} = x - \tilde{x}$
- T=10 s



nonlinear model | linear with forcing | linear without forcing

Results

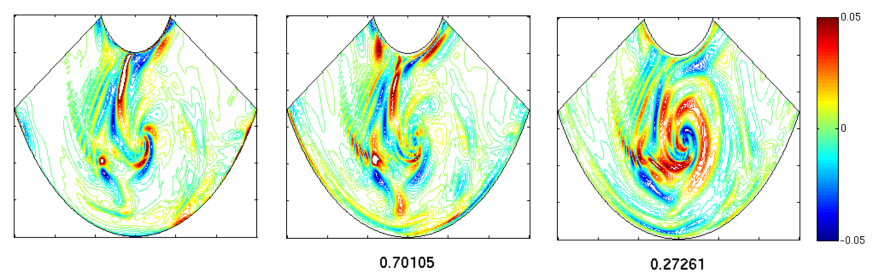
Comparison of horizontal divergence

- Initialising linear model with $x'_{init} = x - \tilde{x}$
- T=20 s

Results

Comparison of horizontal divergence

- Initialising linear model with $x'_{init} = x - \tilde{x}$
- T=20 s



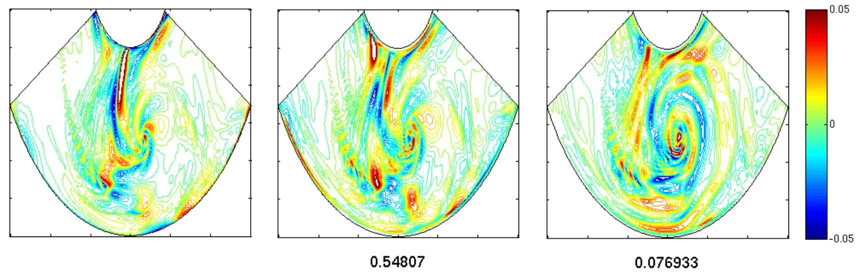
nonlinear model linear with forcing linear without forcing

Comparison of horizontal divergence

- Initialising linear model with $x'_{init} = x - \tilde{x}$
- T=30 s

Comparison of horizontal divergence

- Initialising linear model with $x'_{init} = x - \tilde{x}$
- T=30 s



nonlinear model linear with forcing linear without forcing

Conclusions and Outlook

Conclusion:

- Increased GW activity within the baroclinic wave and close to the inner cylinder wall
- Tangent linear analysis to gain further understanding of the GW source mechanism
- Window function to suppress growth rate at side walls
- Significant internal forcing of GW by the balanced flow

Outlook:

- Extract balanced part of horizontal divergence: omega equation
- Characterizing wave properties $(\vec{k}, A, ..)$

References

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Additional material

Tangent linear annulus equations

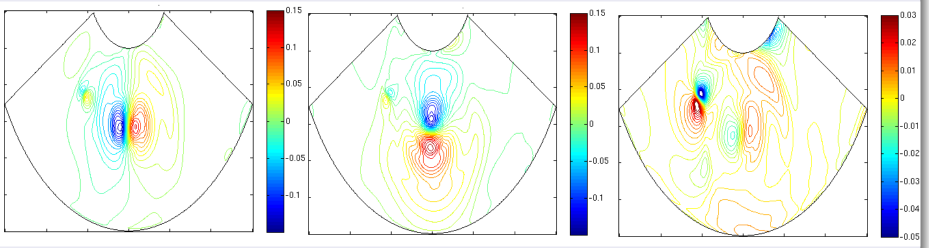
$$\frac{dB_a}{dt} = -N^2 w_a - \left(\frac{dB_g}{dt}\right)_a - \left\{\left(\frac{dB_g}{dt}\right)_g\right\}$$

$$\frac{d\boldsymbol{u_a}}{dt} = -f\,\boldsymbol{e_z} \times \boldsymbol{u_a} - \nabla_h \tilde{p}_{aa} - \left(\frac{d\boldsymbol{u_g}}{dt}\right)_a - \left\{\nabla_h \tilde{p}_{ag} + \left(\frac{d\boldsymbol{u_g}}{dt}\right)_g\right\}$$

$$\frac{dw_a}{dt} = B_a - \frac{\partial \tilde{p}_{aa}}{\partial z} - \left\{\frac{\tilde{p}_{ag}}{\partial z}\right\}$$

Forcing terms

- **Large scale** balanced forcing leading contributor to gravity wave activity
- T=0 s



forcing of u   forcing of v   forcing of b

Additional material

Balanced part of horizontal divergence: Omega equation

- Total horizontal divergence includes balanced part

$$\delta_{total} = \delta_{unbal} + \delta_{bal}$$

- Subtract balanced part using omega equation (Hoskins et al. (1978), Danioux et al. (2012))

$$\begin{aligned} \Rightarrow \delta_{unbal} &= \delta_{total} - \delta_{bal} \\ &= \nabla_h \cdot \boldsymbol{u_a} - \delta_{bal}, \text{ with } \delta_{bal} = -\frac{\partial w_{bal}}{\partial z} \\ \nabla_h^2 w_{bal} &= -\frac{2}{N^2} \nabla_h \cdot \boldsymbol{Q}, \text{ with } \boldsymbol{Q} = \nabla_h \boldsymbol{u_g} \cdot \nabla_h b_g \end{aligned}$$

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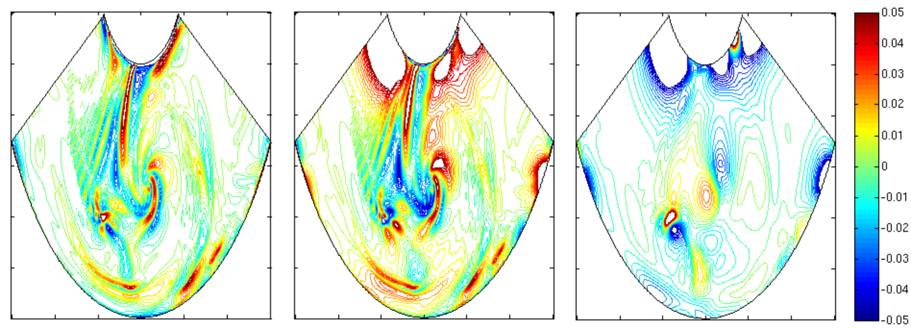

δ_{total} δ_{unbal} δ_{bal}