Gravity wave emission and propagation in the differentially heated rotating annulus experiment

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2016 SPARC Gravity Wave Symposium

05/17/2016
Introduction and Motivation

- Orographic and convective sources of inertia-gravity waves (IGW) well understood (Fritts (2003))
- Spontaneously emitted GWs active field of research (Cámara and Lott (2015), ...)
- Observation identify increased IGW activity in jet exit regions (e.g. Plougonven and Zhang (2014))
- GW emission embedded in various atmospheric processes
- Need for controllable, repeatable and simplified laboratory experiments: **rotating annulus**

Koch and OHandley (1997)
Introduction and Motivation

- Differentially heated rotating annulus experiment:

- Finite volume code (cylFloit) to simulate experiment using Boussinesq approximation (Borchert et al. (2015))
Annulus simulations show clear GW activity (Borchert et al. (2014))
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Further understanding of GW source processes: tangent linear analysis (see Snyder et al. (2009), Wang and Zhang (2010))

⇒ Is there some internal forcing of GWs by the balanced flow?
Decomposition of flow into **geostrophic** and **ageostrophic** part

\[ \mathbf{v} = u_g + v_a \]
\[ B = B_g + B_a \]
\[ p = p_g + p_a \]
Source mechanism of gravity wave emission

Decomposition of flow into **geostrophic** and **ageostrophic** part

\[ \mathbf{v} = \mathbf{u}_g + \mathbf{v}_a \]
\[ B = B_g + B_a \]
\[ p = p_g + p_a \]

\[ f e_z \times \mathbf{u}_g + \nabla_h p_g = 0 \]
\[ B_g - \frac{\partial p_g}{\partial z} = 0 \]

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\[ \Pi_g = \zeta + \frac{f}{N^2} \frac{\partial B}{\partial z} = \frac{1}{f} \left( \nabla_h^2 + \frac{f^2}{N^2} \frac{\partial^2}{\partial z^2} \right) p_g \]

\[ \zeta_a + \frac{f}{N^2} \frac{\partial B_a}{\partial z} = 0 \]

\[ \delta = \nabla_h \cdot \mathbf{u} = \delta_a = \nabla_h \cdot \mathbf{u}_a \]
Source mechanism of gravity wave emission

Decomposition of flow into \textbf{geostrophic} and \textbf{ageostrophic} part

\[ f e_z \times u_g + \nabla_h p_g = 0 \]
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\[ \delta = \nabla_h \cdot \mathbf{u} = \delta_a = \nabla_h \cdot \mathbf{u}_a \]

... \Rightarrow \textbf{Geostrophic forcing of ageostrophic flow}

\[ \frac{D \delta_a}{Dt} = - \frac{\partial B_a}{\partial z} + \frac{\partial^2 p_{aa}}{\partial z^2} + \frac{\partial \mathbf{v}}{\partial z} \cdot \mathbf{w}_a - \frac{\partial^2}{\partial z^2} \nabla^{-2} \left( \nabla u_g \cdot \nabla u_g \right) \]
Tangent Linear Analysis

**Linearisation of unbalanced flow about balanced flow: Principle**

- Decomposition into balanced (large) and unbalanced (small) part

\[ x = \tilde{x} + x' \text{ with } |x'| \ll |\tilde{x}| \quad (\text{Unbalanced part} \equiv \text{gravity waves}) \]

- Tangent linear evolution of \( x' \)

\[
\frac{\partial x'}{\partial t} = L(\tilde{x})x' + F(\tilde{x})
\]

with a linear operator \( L(\tilde{x}) \) and a balanced forcing term \( F(\tilde{x}) \)

- Unbalanced component is integrated separately within each time step
- Balanced part serves as background of the tangent linear model
Tangent Linear Analysis

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**Balanced forcing leading contributor to the gravity wave activity?**
Problem: Instabilities at side walls

- Linear model diverges after about 4-5 s of integration time
- Exponential growth rate at outer (and inner) side walls

*Vertical velocity $w$, Full Forcing, initzero, $T=4s$*
Suppress growth at side wall: Multiplication with window function

\[ f(x) = \begin{cases} 
1, & |x| \leq \beta L_y \\
\frac{1}{2} \left\{ 1 + \cos \left[ \pi \left( \frac{|x| - \beta L_y}{L_s} \right) \right] \right\}, & \beta L_y < |x| \leq [\beta + \gamma (1 - \beta)] L_y \\
0, & \text{else}
\end{cases} \]
Results

Comparison of horizontal divergence

- Initialising linear model with zero unbalanced part
- Full forcing
- $T = 22\,s$ ($\Omega = 0.08\,\text{rad/s}$, rotation period $\sim 79\,s$)
Comparison of horizontal divergence

- Initialising linear model with zero unbalanced part
- Full forcing
- T=22 s (\(\Omega = 0.08 \text{ rad/s}\), rotation period \(\sim 79 \text{ s}\))
Comparison of horizontal divergence

- Initialising linear model with $x'_{init} = x - \tilde{x}$
- $T=0$ s
Results

Comparison of horizontal divergence

- Initialising linear model with $x'_{\text{init}} = x - \tilde{x}$
- $T=0$ s

nonlinear model  linear with forcing  linear without forcing
Comparison of horizontal divergence

- Initialising linear model with $x'_{init} = x - \tilde{x}$
- $T=10$ s
Comparison of horizontal divergence

- Initialising linear model with $x'_{\text{init}} = x - \tilde{x}$
- $T=10$ s

nonlinear model  linear with forcing  linear without forcing
Results

Comparison of horizontal divergence

- Initialising linear model with $x_{\text{init}}' = x - \tilde{x}$
- $T=20$ s
Results

Comparison of horizontal divergence

- Initialising linear model with $x'_{init} = x - \bar{x}$
- $T=20$ s

Nonlinear model
Linear with forcing
Linear without forcing
Comparison of horizontal divergence

- Initialising linear model with $x'_{\text{init}} = x - \tilde{x}$
- $T=30 \text{ s}$
Comparison of horizontal divergence

- Initialising linear model with $x'_{init} = x - \tilde{x}$
- $T=30$ s

![Comparison of horizontal divergence](image.png)

- Nonlinear model
- Linear with forcing
- Linear without forcing
Conclusion:

- Increased GW activity within the baroclinic wave and close to the inner cylinder wall
- Tangent linear analysis to gain further understanding of the GW source mechanism
- Window function to suppress growth rate at side walls
- Significant internal forcing of GW by the balanced flow

Outlook:

- Extract balanced part of horizontal divergence: omega equation
- Characterizing wave properties ($\vec{k}, A, ..$)
References


Tangent linear annulus equations

\[
\frac{dB_a}{dt} = - N^2 w_a - \left( \frac{dB_g}{dt} \right)_a - \left\{ \left( \frac{dB_g}{dt} \right)_g \right\}
\]

\[
\frac{du_a}{dt} = - f e_z \times u_a - \nabla_h \tilde{\rho}_{aa} - \left( \frac{du_g}{dt} \right)_a - \left\{ \nabla_h \tilde{\rho}_{ag} + \left( \frac{du_g}{dt} \right)_g \right\}
\]

\[
\frac{dw_a}{dt} = B_a - \frac{\partial \tilde{\rho}_{aa}}{\partial z} - \left\{ \frac{\tilde{\rho}_{ag}}{\partial z} \right\}
\]
Forcing terms

- **Large scale** balanced forcing leading contributor to gravity wave activity
- \( T = 0 \ s \)
Balanced part of horizontal divergence: Omega equation

- Total horizontal divergence includes balanced part

\[ \delta_{\text{total}} = \delta_{\text{unbal}} + \delta_{\text{bal}} \]

- Subtract balanced part using omega equation (Hoskins et al. (1978), Danioux et al. (2012))

\[ \Rightarrow \delta_{\text{unbal}} = \delta_{\text{total}} - \delta_{\text{bal}} \]

\[ = \nabla_h \cdot \mathbf{u}_a - \delta_{\text{bal}}, \text{ with } \delta_{\text{bal}} = -\frac{\partial w_{\text{bal}}}{\partial z} \]

\[ \nabla_h^2 w_{\text{bal}} = -\frac{2}{N^2} \nabla_h \cdot \mathbf{Q}, \text{ with } \mathbf{Q} = \nabla_h \mathbf{u}_g \cdot \nabla_h b_g \]
Balanced part of horizontal divergence: Omega equation

\[ \delta_{total} \quad \delta_{unbal} \quad \delta_{bal} \]