Comments on "Application of the Lighthill–Ford Theory of Spontaneous Imbalance to Clear-Air Turbulence Forecasting"

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1. Introduction

Knox et al. (2008, hereafter KMW) document an empirical relation between a diagnostic quantity and clear-air turbulence (CAT) and argue for a theoretical basis involving gravity waves to support this result. While we do not question the empirical evidence for the effectiveness of this diagnostic, KMW justify its success based on the work of Ford (1994a), who described the emission of gravity waves by balanced vortex motions in rotating shallow water. Ford's work itself was based on the ideas of Lighthill (1952), describing the emission of acoustic waves by small-scale turbulent motions. In fact, that theory is not applicable to the flows and conditions that KMW consider.

2. Diagnosing spontaneous generation of gravity waves in shallow water

KMW argue that "a significant limitation for the forecasting of all types of turbulence is identifying the source of gravity waves" and hence base their approach of CAT forecasting on studies of spontaneous gravity wave generation. Their new indicator finds its origin in a rearrangement of the equations of motion used by Ford (1994a) to predict the emission of gravity waves by balanced vortex motions, and hence KMW interpret these forcing terms as indicative of generation of gravity waves. They further assume that gravity waves play a role in triggering CAT and hence use their "Lighthill–Ford source term" as an indicator for turbulence.

In this reasoning, KMW overlook two important aspects of the shallow water problem analyzed by Ford (1994a): first, there is a spatial scale separation between the large-scale gravity waves in the far field and the small-scale balanced motions generating the waves (section 2a). Second, the forcing terms obtained on the rhs contribute not only to the generation of gravity waves, but also and predominantly to the balanced dynamics (section 2b).

a. Regarding spatial scales

By rearranging the equations of motion so as to make a linear wave operator appear on the lhs, Ford (1994a) obtained an equation of the general form

$$\mathcal{L}(\partial_t h) = \mathcal{N}(u, v, h), \tag{1}$$

where $\mathcal{L}$ is the linear gravity wave operator and $\mathcal{N}$ is a forcing coming from the nonlinear advection terms and their derivatives. In the case of the rotating shallow water model investigated by Ford (1994a), this equation can be written

$$\left(\frac{\partial^2}{\partial t^2} + f^2 - g\,h_0\nabla^2\right)\frac{\partial h}{\partial t} = \frac{\partial^2}{\partial x_i \partial x_j} T_{ij}, \tag{2}$$

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where g is gravity, h is the height of the free surface, h_0 is the height for the fluid at rest, and f is the Coriolis parameter. The compact form of the nonlinear forcing terms on the rhs, $\partial_{x_i x_j} T_{ij}$, is obtained when the equations for momentum and mass conservation are written in flux form.

To predict the emission of gravity waves, ‘‘the key point in Lighthill’s theory is now to assume that the source term T_{ij} is only nonzero over a small enough region that the right-hand side of (2) may be approximated by a quadrupole point source’’ (Ford 1994a). In other words, it is crucial that there is a separation between the spatial scales of the two types of motions. This scale separation is tied to the smallness of the Froude number $F = U/(gh_0)^{1/2} \ll 1$ and the common height scale [see Plougonven and Zeitlin (2002) or Schecter and Montgomery (2006) for an application to the baroclinic case]. In practice, this scale separation allowed Ford to use either a Green’s function (Ford 1994a) or matched asymptotics (Ford 1994b) to obtain the gravity waves in the far field (i.e., in a different region of space than the balanced vortex motions).

Hence, in configurations such as those described by Ford et al. (2000) (rotating shallow water, small Froude number, Rossby number larger than unity), using large values of the rhs forcing term $\mathcal{N}$ as a local indicator of spontaneously generated gravity waves is at the very least misleading: the waves are present in the far field, where $\mathcal{N}$ is zero, and their spatial scales are much larger.

An additional remark can be made regarding this separation of scales, as this is one of the unresolved issues regarding the laboratory experiments of Williams et al. (2005), to which KMW refer as support for their approach. Williams et al. (2005) describe two-layer flows in a rotating annulus, for which short-scale features, interpreted as inertia–gravity waves, appear within certain regions of a baroclinic wave undergoing vacillation. This region coincides with the large-scale maximum of the Lighthill–Ford forcing term. There are at least two crucial unresolved issues: 1) How does this large-scale forcing generate small-scale waves? 2) Why does a quadratic forcing produce waves with amplitude linear in Rossby number? A spatial coincidence between well-identified inertia–gravity waves and a diagnostic of imbalance—for example, the Lagrangian Rossby number (O’Sullivan and Dunkerton 1995; Plougonven et al. 2003) or the residual of the nonlinear balance equation (Zhang 2004)—is of interest, but without a more systematic investigation of the variations of the excited waves relative to the forcing, it only provides an indication, not compelling evidence for a generation mechanism.

b. Regarding time scales

Regardless of the separation of spatial scales, it is of course always possible to rearrange the equations in such a way that one obtains a linear wave operator on the lhs and forcing terms on the rhs as in Eq. (1). Once the equations have been rearranged, ‘‘no profound progress has been made’’ (Snyder et al. 1993): the forcing terms will force both balanced and gravity wave responses. If one’s purpose is to quantify spontaneous emission, the difficult task is then to determine which part of $\mathcal{N}$ produces gravity waves. In other words, there is no direct relationship between large instantaneous values of $\mathcal{N}$ and strong gravity wave generation. To illustrate this, we obtain below a wave equation forced by nonlinear terms in the case of a continuously stratified fluid and show that the well-known omega equation (e.g., Holton 1992) is embedded in it.

We start from the primitive equations for a hydrostatic fluid in the Boussinesq approximation and on the f plane (McWilliams and Gent 1980):

$$D_t \mathbf{u}_H + f\mathbf{k} \times \mathbf{u}_H + \nabla\phi = 0, \tag{3a}$$

$$\phi_z = \chi, \tag{3b}$$

$$D_t \chi + N^2 w = 0, \quad \text{and} \tag{3c}$$

$$u_x + v_y + w_z = 0, \tag{3d}$$

where ϕ is geopotential; z is a pseudoheight (Hoskins and Bretherton 1972); $\chi = g/\theta_0\theta$, with θ being the potential temperature; and N^2 is the square of the Brunt–Väisälä frequency, assumed to be uniform; $D_t = \partial_t + \mathbf{u}\cdot\nabla$ is the full Lagrangian derivative.

We rewrite the equations for horizontal momentum [Eq. (3a)] and for the potential temperature [Eq. (3c)] in a way that isolates quasigeostrophic dynamics (Holton 1992):

$$\mu\partial_t \mathbf{u}_a + D_g \mathbf{u}_g + f\mathbf{k} \times \mathbf{u}_a = -\mu\mathcal{M}_{\mathbf{u}} \quad \text{and} \tag{4a}$$

$$D_g \chi + N^2 w = -\mu\mathcal{M}_\chi, \tag{4b}$$

where $\mathbf{u}_g = f^{-1}\mathbf{k} \times \nabla\phi$ is the geostrophic wind, $\mathbf{u}_a = \mathbf{u} - \mathbf{u}_g$ is the ageostrophic residual, and $D_g = \partial_t + \mathbf{u}_g \cdot \nabla$ includes only the advection by the geostrophic wind. Setting $\mu = 1$ yields the full primitive equations [Eqs. (3)], whereas $\mu = 0$ leads to the quasigeostrophic approximation. The nonlinear terms on the rhs are the advection terms involving $\mathbf{u}_a$:

$$\mathcal{M}_{\mathbf{u}} = \mathbf{u}_a \cdot \nabla\mathbf{u}_g + \mathbf{u} \cdot \nabla\mathbf{u}_a \quad \text{and} \tag{5a}$$

$$\mathcal{M}_\chi = \mathbf{u}_a \cdot \nabla\chi. \tag{5b}$$

Standard manipulations of these lead to

$$\mu\partial_{tt}\partial_{zz}w + f^2\partial_{zz}w + N^2\Delta_H w = -2\nabla\cdot\mathbf{Q} + \mu[f\mathbf{k}\cdot\partial_z(\nabla\times\mathcal{M}_{\mathbf{u}}) - \Delta_H\mathcal{M}_\chi + \partial_{tz}\nabla\cdot\mathcal{M}_{\mathbf{u}}], \tag{6}$$

where $\mathbf{Q} = (\partial_x\mathbf{u}_g \cdot \nabla\chi)\mathbf{i} + (\partial_y\mathbf{u}_g \cdot \nabla\chi)\mathbf{j}$ is the $\mathbf{Q}$ vector (Holton 1992).

Equation (6) is of the same form as Eq. (1). On the lhs, we recognize (for $\mu = 1$) the wave operator for inertia–gravity waves in a fluid at rest (e.g., Holton 1992, chapter 7). On the rhs, one has (for $\mu = 1$) the "forcing" $\mathcal{N}$.

Now, embedded in Eq. (6), one clearly recognizes (for $\mu = 0$) the $\mathbf{Q}$-vector form of the omega equation. This provides, within quasigeostrophic theory, a diagnostic of the vertical motion w. Therefore, solutions to Eq. (6) are not only gravity waves, but also include slow, balanced motions. In fact, for midlatitude flows the balanced part of the vertical motion will generally dominate (e.g., Viúdez and Dritschel 2006). Nonzero values of the forcing terms $\mathcal{N}$ do not systematically indicate gravity wave generation, and it is therefore wrong to interpret these simply as a "source of gravity waves."

Although KMW do not discuss it, this is the essential difficulty of the problem of spontaneous generation: identifying the small fraction of $\mathcal{N}$ that *does* project onto gravity waves rather than slow motions. In support for their approach, KMW refer to Medvedev and Gavrilov (1995), who seek to diagnose gravity wave sources using asymptotic, multi-time-scale expansions. But these authors also evade this difficulty as they relax their multi-time-scale assumption so as to obtain a tentative "gravity wave source." Without relaxing this assumption, their approach would on the contrary lead to the conclusion that no spontaneous generation occurs (Reznik et al. 2001; Zeitlin et al. 2003) for the flows described (small Rossby number; one unique length scale). Because Medvedev and Gavrilov (1995) do not provide any evidence for the relevance of their gravity wave source—for example, in the form of full primitive equation simulations of synoptic flows exhibiting spontaneous generation (O'Sullivan and Dunkerton 1995; Zhang 2004; Plougonven and Snyder 2007)—their work cannot be considered as support for KMW.

3. Discussion

The previous section shows that the "Lighthill–Ford source term" cannot be used as a local indicator of spontaneously generated gravity waves even in rotating shallow water where the Lighthill–Ford theory is valid. Now, as they acknowledge, KMW are interested in continuously stratified applications, where Lighthill–Ford theory need not apply. Indeed, the phenomenology of gravity waves spontaneously generated by jets and fronts in both observations (e.g., Thomas et al. 1999; Pavelin et al. 2001; Zülicke and Peters 2006) and simulations (e.g., O'Sullivan and Dunkerton 1995; Zhang 2004; Plougonven and Snyder 2007) is distinctly different than predicted by that theory because the waves are small scale relative to the flow generating them, not large scale.

This fact has important implications for understanding the generation of the waves. For waves that are small scale relative to a nonzero background flow, the advection by this flow becomes essential for determining the characteristics of the waves. The relevant lhs operator in Eq. (1) no longer is the operator for waves in a fluid at rest but should have varying coefficients (Plougonven and Zhang 2007). In general, the properties of a linear wave operator on the background of a complex, three-dimensional, time-dependent flow will not be easily known. Moreover, even for simple flows the large-scale flow relative to the forcing will be crucial to determine the response (as for mountain waves). This makes it very difficult to interpret the forcing term on its own in contrast to the configuration of Ford's problem, where a temporal Fourier transform of $\mathcal{N}$ was sufficient to isolate the part of $\mathcal{N}$ that contributes to the generation of gravity waves [e.g., Eqs. (9) and (13) in Ford (1994a)].

Last, we can note that continuously stratified flows allow propagation effects such as *wave capture* (Badulin and Shrira 1993; Bühler and McIntyre 2005) that are excluded in shallow water. This mechanism was shown to be relevant for waves generated by a jet in baroclinic life cycle experiments (Plougonven and Snyder 2005) and in dipoles (Snyder et al. 2007; Viúdez 2007; Wang et al. 2009). The opposite separation of scales (small-scale waves in a large-scale flow, not the reverse) and the possibility of wave capture make the problem of spontaneously generated waves by jets and fronts (e.g., O'Sullivan and Dunkerton 1995) fall outside the scope of Lighthill–Ford theory, as clearly explained by McIntyre (2009).

4. Conclusions

In conclusion, it is worth recalling that 1) the new turbulence indicator introduced by KMW may well be very efficient and relevant for CAT and 2) the above arguments do not exclude the possibility that gravity waves can play a role in triggering CAT. Case studies have shown it is possible (e.g., Lane et al. 2004; Koch et al. 2005).

However, we take issue with KMW's implicit claim to have applied Lighthill–Ford theory (Lighthill 1952; Ford et al. 2000) to predict the location and intensity of spontaneous gravity wave generation in midlatitude flow. The Lighthill–Ford theory is valid in different regimes than those considered by KMW and even when valid does not provide a spatially local predictor for spontaneously emitted waves. Moreover, it makes predictions that are qualitatively at odds with observations and simulations of inertia–gravity waves generated by midlatitude jets and fronts. Hence, we argue that KMW's interpretation of their CAT indicator $\mathcal{N}$ as involving gravity waves is not founded and that their study does not bring any element to the debates on the generation of gravity waves or on their role in producing CAT. Only a fraction of $\mathcal{N}$ will contribute to the generation of gravity waves, and KMW evade this difficulty. On the other hand, $\mathcal{N}$ is certainly relevant as an indicator of strong forcing of (mostly balanced) vertical motions and of significant nonlinearity in the flow, as shown by Eq. (6). Balanced motions include frontogenesis, which leads to regions of very strong shears where small-scale shear instabilities likely become important (e.g., Snyder 1995). Hence, it is not necessary to invoke gravity waves to see how $\mathcal{N}$ could be correlated to regions of CAT. In addition, as discussed by KMW, $\mathcal{N}$ is related to several established indicators for CAT, providing further reason to expect that $\mathcal{N}$ should be correlated with CAT regardless of its dynamical underpinnings.

Thus, the effectiveness of $\mathcal{N}$ as a CAT indicator is likely due to reasons other than those put forward in KMW. A more rigorous interpretation will be needed ''to place the subject of CAT forecasting on a firmer theoretical footing'' (KMW).

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