NOTES AND CORRESPONDENCE

On the Forcing of Inertia–Gravity Waves by Synoptic-Scale Flows

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ABSTRACT

Studies on the spontaneous emission of gravity waves from jets, both observational and numerical, have emphasized that excitation of gravity waves occurred preferentially near regions of imbalance. Yet a quantitative relation between the several large-scale diagnostics of imbalance and the excited waves is still lacking.

The purpose of the present note is to investigate one possible way to relate quantitatively the gravity waves to diagnostics of the large-scale flow that is exciting them. Scaling arguments are used to determine how the large-scale flow may provide a forcing on the right-hand side of a wave equation describing the linear dynamics of the excited waves. The residual of the nonlinear balance equation plays an important role in this forcing.

1. Introduction

Gravity waves are of importance in the atmosphere for vertical transfers of momentum toward the middle atmosphere (Fritts and Alexander 2003). As subgrid-scale phenomena, they are generally parameterized in general circulation models, but their sources, in particular jets and fronts, remain insufficiently constrained (Kim et al. 2003). A first attempt to parameterize jet/front systems as sources of gravity waves was presented by Charron and Manzini (2002), who simply related the excitation of the gravity waves to regions of intense frontogenesis. However, this was based on qualitative arguments. A quantitative understanding of the mechanisms generating gravity waves from jets and fronts is still lacking.

Now, it has been repeatedly noted in both observational (Uccelini and Koch 1987; Plougonven et al. 2003) and numerical studies (O'Sullivan and Dunkerton 1995; Zhang et al. 2001; Zhang 2004) that inertia–gravity waves (IGWs) appeared in the vicinity of unbalanced regions. These regions can be obtained from the large-scale flow, for instance from the European Centre for Medium-Range Weather Forecasts (ECMWF) analyses (Plougonven et al. 2003), using various diagnostics of imbalance: the Lagrangian Rossby number (O'Sullivan and Dunkerton 1995), the cross-stream Lagrangian Rossby number (Koch and Dorian 1988), or the residual of the nonlinear balance equation (NBE; see Zhang et al. 2001 and references therein). Yet, the use of these large-scale diagnostics is only based on qualitative arguments: the evolution of the jet/front system produces a region of imbalance that should undergo "geostrophic" or "balance" (Zhang 2004) adjustment, thereby producing gravity waves.

Following the ideas of Lighthill on sound generation by turbulence (Lighthill 1952), Ford (1994a; Ford et al. 2000) suggested to rewrite the equations of motion in such a way that we obtain a linear wave equation (for waves in a fluid at rest) on the left-hand side and a forcing due to nonlinear terms linked to the vortical (balanced) motions on the right-hand side. It was shown that this could provide an efficient way to pre-

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dict large-scale waves emitted in the far field (Ford 1994a,b; Plougonven and Zeitlin 2002). In a context more comparable with the atmosphere, a similar approach was applied to the problem of IGWs excited by 2D frontogenesis by Reeder and Griffiths (1996). One notable difference with the work of Ford (1994a) was that the wave equation included the large-scale background flow of deformation and shear, which considerably influenced the characteristics of the emitted waves. Another example showing the importance of propagation for the characteristics of spontaneously emitted gravity waves was given by Plougonven and Snyder (2005). Recently, laboratory experiments (Williams et al. 2005) have also suggested that the approach proposed by Ford should be relevant for locating sources of IGWs within a two-layer fluid.

In the present note, our motivation is to provide a basis for a quantitative relationship between some diagnostic of imbalance and the amplitude of the IGWs excited. We will use scaling arguments to identify candidates for what the wave equation and what the forcing terms from the large-scale flow should be. In section 2 we will briefly discuss the issue of the time separation between the balanced flow and the gravity waves. The scaling of the equations will be presented in section 3. A form for a wave equation and for the corresponding forcing terms will be obtained in section 4. Conclusions and perspectives are discussed in section 5.

2. Description of the different components of motions

To obtain an equation for waves forced by the large-scale, mostly balanced motions, it is necessary to separate the flow into these two components. Below we review three issues regarding this separation: the time-scale separation, the difference between Eulerian and Lagrangian time scales, and finally the separation of the flow.

An essential feature of the dynamics of midlatitude flows is the time-scale separation between the large-scale balanced flow (a few days) and the inertia–gravity waves (several hours; Gill 1982). This separation is part of the reason why interactions between the two types of motions are weak (Reznik et al. 2001; Ford et al. 2000, 2002; Saujani and Shepherd 2002). This time-scale separation is not valid everywhere, as it breaks down locally where the flow generates small scales, for example, in regions of frontogenesis.

If one separates formally the two time scales using standard multi-timescale expansions, a weak excitation of gravity wave motions from jets and fronts cannot be detected: indeed, forcing from the primary flow will necessarily be slow, and hence cannot force a fast response (Reznik et al. 2001; Zeitlin et al. 2003), unless one precisely relaxes the assumption of time-scale separation (Medvedev and Gavrilov 1995).

A second point to bear in mind regarding time scales is the key role of the Lagrangian time scale: an evident illustration is given by gravity waves in a steady flow above a mountain ridge. Another illustration is the transition from balanced to gravity wave motions through the inertial critical layer in a vertical shear (Plougonven et al. 2005).

Hence in our approach we will avoid separating time scales rigidly between the primary and secondary flows and will allow for the possibility that advection by the primary flow matters for the secondary flow.

Now, the flow needs to be separated into two different components. Splitting the flow into a balanced part and an unbalanced part could be an evident choice, as was discussed in Snyder et al. (1993). However, we prefer to leave the precise definition of the separation open: having in mind applications to parameterizations in general circulation models (GCMs), we would rather have a coarse-grained description of the full flow, which is not fully balanced, and would like to estimate smaller-scale elements of the flow.

3. Scaling separating the two components of the flow

The primitive equations used are those for a fluid that is adiabatic, inviscid, and hydrostatic within the Boussinesq approximation and on the β plane (McWilliams and Gent 1980):

$$\partial_t \mathbf{u}_H^* + \mathbf{u}^* \nabla \mathbf{u}_H^* + f \mathbf{e}_z \times \mathbf{u}_H^* + \nabla_H \Phi^* = 0, \tag{1a}$$

$$-\frac{g}{\theta_0} \theta^* + \partial_z \Phi^* = 0, \tag{1b}$$

$$\partial_t \theta^* + \mathbf{u}^* \nabla \theta^* = 0, \quad \text{and} \tag{1c}$$

$$\nabla \cdot \mathbf{u}^* = 0, \tag{1d}$$

where * denotes dimensional variables, $f = f_0 + \beta y$ is the Coriolis parameter, θ_0 is a reference potential temperature, g is the gravitational acceleration, Φ^* is the geopotential height, θ^* is potential temperature, and z is a modified pressure coordinate (Hoskins and Bretherton 1972), which we will refer to simply as height. The subscript H indicates the horizontal component of a vector and $\mathbf{e}_z$ is the unit vertical vector. The excited IGWs are known to be of rather low frequency

(Plougonven et al. 2003), justifying the hydrostatic balance. The Boussinesq approximation is made, assuming that this will be sufficient, in a first approach, to identify the form of the wave equation and forcing terms that interest us.

Now, the first two equations are rewritten in order to have an equation for the divergence of the horizontal wind, $\delta^* = \partial_x u^* + \partial_y v^*$, and one for the relative vorticity $\zeta^* = \partial_x v^* - \partial_y u^*$. Hence we obtain

$$\partial_t \delta^* + \mathbf{u}^* \nabla \delta^* + \mathbf{u}_x^* \nabla u^* + \mathbf{u}_y^* \nabla v^* - f\zeta^* + \beta u^* + \Delta_H \Phi^* = 0 \quad \text{and} \tag{2a}$$

$$\partial_t \zeta^* + \mathbf{u}^* \nabla \zeta^* + \mathbf{u}_x^* \nabla v^* - \mathbf{u}_y^* \nabla u^* + f\delta^* + \beta v^* = 0. \tag{2b}$$

It will be useful to rewrite some of the above terms as follows:

$$\mathbf{u}_x^* \nabla u^* + \mathbf{u}_y^* \nabla v^* = 2\mathcal{J}(v^*, u^*) + \delta^{*2} + (\partial_z \mathbf{u}_H^*)\nabla w^* \quad \text{and} \tag{3a}$$

$$\mathbf{u}_x^* \nabla v^* - \mathbf{u}_y^* \nabla u^* = \zeta^* \delta^* + (\mathbf{e}_z \times \nabla w^*) \cdot (\partial_z \mathbf{u}_H^*), \tag{3b}$$

where $\mathcal{J}$ is the Jacobian operator $\mathcal{J}(v, u) = v_x u_y - u_x v_y$. Now, we will consider that there are two types of motions: $u^* = \overline{u}^* + u^{*\prime}$. The "primary" flow (noted with a bar: $\overline{u}^*$) is essentially balanced (nondivergent) and large-scale. The "secondary" flow (noted with a prime: $u^{*\prime}$) is smaller in amplitude, is not balanced, and can have smaller scales.

The primary fields are scaled taking into account that their horizontal divergence is weak: $\overline{u}^*$ and $\overline{v}^*$ are of order U, whereas $\overline{w}^*$ is only of order $\epsilon UH/L$, with $\epsilon \ll 1$. The time scale is the advective time scale, of order L/U. We will assume that $\beta L/f = \epsilon$ and that the Rossby and Burger numbers verify

$$\epsilon = \frac{U}{fL} \ll 1 \quad \text{and} \quad \frac{f^2 L^2}{N^2 H^2} = 1. \tag{4}$$

The secondary fields are smaller in amplitude than the primary fields: the velocities $u^{*\prime}$ and $v^{*\prime}$ scale as αU, with $\alpha \ll 1$. The secondary fields can vary on smaller scales, γL in the horizontal and γH in the vertical, with $\gamma \leq 1$. From the polarization relations for internal gravity waves, the geopotential anomaly scales as $\alpha f U \gamma L$. Based on experience from numerical and observational studies we expect the frequencies of excited gravity waves to be at the lower end of the frequency spectrum for inertia–gravity waves. Hence their time scale is scaled as f^{-1}. Finally, as their horizontal divergence is not particularly weak, $w^{*\prime}$ scales as $\alpha UH/L$.

Now, rewriting the equations with this separation produces terms that can be grouped into

- terms that only involve the primary fields, noted $\mathcal{A}^*$
- terms that are linear in the secondary fields, noted $\mathcal{L}^*$
- terms that are quadratic in the secondary fields, noted $\mathcal{Q}^*$

The scaling of all these terms will then indicate which terms in $\mathcal{L}^*$ need to be retained to build a wave equation and which terms in $\mathcal{A}^*$ need to be retained to provide a forcing.

Two choices for the relative scales of the primary and secondary motions will be considered (parameter γ). These imply different choices for the amplitude of the secondary motions (parameter α):

1) The simplest choice corresponds to waves that have spatial scales comparable to that of the primary flow: $\gamma = 1$. We then simply choose $\alpha = \epsilon$, indicating that the waves are one order of magnitude smaller than the primary flow.
2) The second and more relevant choice describes waves that have spatial scales smaller than those of the primary flow: $\gamma = \epsilon \ll 1$; such choice implies that the spatial derivatives of the secondary fields scale as $1/\epsilon$ times the derivatives of the primary field. As we expect that the dominant terms in the equations are still the ones associated to the primary flow, we then have to choose $\alpha = \epsilon^2$. Hence for both choices of relative scales we have $\alpha/\gamma = \epsilon$.

In the three subsections below, nondimensional variables will be denoted by the absence of a * superscript; for example, $\overline{u}$ and u'.

a. Scaling of the divergence equation

When the divergence Eq. (2a) is rewritten with all fields separated into a slow and a fast part, the scaling then divides the equation into

$$\epsilon^{-1}\mathcal{A}_{0,\delta} + \mathcal{A}_{1,\delta} + \mathcal{L}_{1,\delta} + \epsilon\mathcal{A}_{2,\delta} + \epsilon\mathcal{L}_{2,\delta} + \epsilon^2 \mathcal{Q}_\delta = 0, \tag{5}$$

where the different groups of terms are

$$\mathcal{A}_{0,\delta} = -\overline{\zeta} + \Delta_H \overline{\phi}, \quad \mathcal{A}_{1,\delta} = 2\mathcal{J}(\overline{v}, \overline{u}) + \overline{u}, \tag{6a}$$

$$\mathcal{L}_{1,\delta} = \partial_t \delta' + \epsilon\gamma^{-1} \overline{\mathbf{u}}_H \nabla \delta' - \zeta' + \Delta_H \phi', \tag{6b}$$

$$\mathcal{A}_{2,\delta} = (\partial_t + \overline{\mathbf{u}}_H \nabla + \epsilon \overline{w} \partial_z)\overline{\delta} + (\partial_z \overline{\mathbf{u}}_H)\nabla \overline{w} + \epsilon \overline{\delta}^2, \tag{6c}$$

$$\mathcal{L}_{2,\delta} = \partial_x(\overline{\mathbf{u}}_H + \epsilon\overline{w}\mathbf{e}_z)\nabla u' + \partial_y(\overline{\mathbf{u}}_H + \epsilon\overline{w}\mathbf{e}_z)\nabla v' + \mathbf{u}_x' \nabla \overline{u} + \mathbf{u}_y' \nabla \overline{v} + \epsilon\gamma^{-1}\overline{w}\partial_z \delta' + \epsilon\gamma \mathbf{u}' \nabla \overline{\delta} + \gamma u', \quad \text{and} \tag{6d}$$

$$\mathcal{Q}_\delta = \mathbf{u}' \nabla \delta' + \mathbf{u}_x' \nabla u' + \mathbf{u}_y' \nabla v'. \tag{6e}$$

The leading order terms involving only the primary fields ($\mathcal{A}_{0,\delta}$) are larger by an order of magnitude than the leading order terms involving the secondary fields. Nevertheless, it is reasonable to assume that their combination, expressing geostrophic equilibrium, is one order smaller than the individual terms.

Among the terms that are linear in the secondary fields, the advective derivative involving the primary horizontal velocity is present if $\gamma \ll 1$. Hence, retaining only leading-order terms to form an equation for the secondary fields, and returning to dimensional variables, we obtain

$$D_\gamma \delta^{*\prime} - f\zeta^{*\prime} + \Delta\phi^{*\prime} = \overline{\zeta}^* - \Delta_H \overline{\phi}^* - \beta\overline{u}^* - 2\mathcal{J}(\overline{v}^*, \overline{u}^*) = \Delta_{\text{NBE}}, \tag{7}$$

where Δ_{NBE} is the residual of the nonlinear balance equation (Zhang et al. 2000) and where we note

$$D_\gamma = \partial_t \quad \text{if} \quad \gamma = 1 \quad \text{and} \tag{8a}$$

$$D_\gamma = \partial_t + \overline{\mathbf{u}}_H^* \boldsymbol{\nabla} \quad \text{if} \quad \gamma = \epsilon \ll 1. \tag{8b}$$

b. Scaling of the vorticity equation

Proceeding as above to scale the vorticity Eq. (2b), the following is obtained:

$$\mathcal{A}_{1,\zeta} + \mathcal{L}_{1,\zeta} + \epsilon\mathcal{A}_{2,\zeta} + \epsilon\mathcal{L}_{2,\zeta} + \epsilon^2 Q_\zeta = 0, \tag{9}$$

where

$$\mathcal{A}_{1,\zeta} = \partial_t\overline{\zeta} + \overline{\mathbf{u}}_H\boldsymbol{\nabla}\overline{\zeta} + \overline{\delta} + \overline{v}, \tag{10a}$$

$$\mathcal{L}_{1,\zeta} = \partial_t\zeta' + \epsilon\gamma^{-1}\overline{\mathbf{u}}_H\boldsymbol{\nabla}\zeta' + \delta', \tag{10b}$$

$$\mathcal{A}_{2,\zeta} = \overline{w}\partial_z\overline{\zeta} + \overline{\delta}\,\overline{\zeta} + (\partial_z\overline{\mathbf{u}}_H)(\mathbf{e}_z \times \boldsymbol{\nabla}\overline{w}), \tag{10c}$$

$$\mathcal{L}_{2,\zeta} = \epsilon\gamma^{-1}\overline{w}\partial_z\zeta' + \gamma\mathbf{u}'\boldsymbol{\nabla}\overline{\zeta} + \gamma v' + \overline{\mathbf{u}}_x\boldsymbol{\nabla}v' - \overline{\mathbf{u}}_y\boldsymbol{\nabla}u' + \mathbf{u}'_x\boldsymbol{\nabla}\overline{v} - \mathbf{u}'_y\boldsymbol{\nabla}\overline{u}, \quad \text{and} \tag{10d}$$

$$Q_\zeta = \mathbf{u}'\boldsymbol{\nabla}\zeta' + \mathbf{u}'_x\boldsymbol{\nabla}v' - \mathbf{u}'_y\boldsymbol{\nabla}u'. \tag{10e}$$

Again if we retain only leading-order terms, we obtain in dimensional variables

$$D_\gamma\zeta^{*\prime} + f\delta^{*\prime} = -\mathcal{A}_{1,\xi^*} = -\partial_t\overline{\zeta}^* - \overline{\mathbf{u}}^*\boldsymbol{\nabla}\overline{\zeta}^* - f\overline{\delta}^* - \beta\overline{v}^*. \tag{11}$$

c. Scaling of the potential temperature equation

For the equation of the potential temperature, it is preferable to isolate the background stratification. The potential temperature field is thus written $\theta^* = \Theta^*(z) + \overline{\theta}^* + \theta^{*\prime}$. From the scalings chosen above for the other variables, and using hydrostaticity, it is found that $\theta^{*\prime}$ will scale as α relative to $\overline{\theta}^*$. The following is then obtained:

$$\epsilon\alpha^{-1}\mathcal{A}_{0,\theta} + \mathcal{L}_{1,\theta} + \epsilon\mathcal{L}_{2,\theta} + \epsilon^2 Q_\theta = 0, \tag{12}$$

where

$$\mathcal{A}_{0,\theta} = \partial_t\overline{\theta} + (\overline{\mathbf{u}}_H\boldsymbol{\nabla} + \epsilon\overline{w}\partial_z)\overline{\theta} + \overline{w}\partial_z\Theta, \tag{13a}$$

$$\mathcal{L}_{1,\theta} = \partial_t\theta' + \epsilon\gamma^{-1}\overline{\mathbf{u}}_H\boldsymbol{\nabla}\theta' + w'\partial_z\Theta, \quad \text{and} \tag{13b}$$

$$\mathcal{L}_{2,\theta} = \epsilon\gamma^{-1}\overline{w}\partial_z\theta' + \overline{u}\boldsymbol{\nabla}\overline{\theta}, \quad Q_\theta = \mathbf{u}'\boldsymbol{\nabla}\theta'. \tag{13c}$$

The scaling of the potential temperature Eq. (1c) differs from that of the other equations: whereas $\zeta^{*\prime}$ scaled as $\alpha\gamma^{-1} = \epsilon$ of $\overline{\zeta}^*$, $\theta^{*\prime}$ scales as α of $\overline{\theta}^*$. As a result, α appears in Eq. (12). In the case where the waves have smaller scales than the primary fields we will assume, as previously, that although the terms involving the primary fields ($\mathcal{A}_{0,\theta}$) are individually larger than the leading-order terms involving the secondary fields, their combination is an order of magnitude smaller than the individual terms. Retaining only the dominant terms, we obtain in dimensional variables

$$D_\gamma\theta^{*\prime} + w^{*\prime}\partial_z\Theta^* = -\mathcal{A}_{0,\theta^*} = -(\partial_t + \overline{\mathbf{u}}^*\boldsymbol{\nabla})\overline{\theta}^* - \overline{w}^*\partial_z\Theta^*. \tag{14}$$

The forcing term here describes the conservation of the potential temperature for the primary fields. Hence, for definitions of the primary field, which respect the conservation of potential temperature, the contribution of this term disappears.

4. Forcing of gravity waves by the large-scale flow

Following scaling arguments, an equation that is linear in the secondary fields and forced by terms from the primary field has been obtained for the divergence [Eq. (7)], for the vorticity [Eq. (11)], and for the potential temperature [Eq. (14)]. These can now be combined to obtain a wave equation for the vertical velocity $w^{*\prime}$ that is forced by terms from the primary flow.

If we consider the case where the waves have scales comparable with those of the primary flow ($\gamma = 1$), the linear operators on the rhs of Eqs. (7), (11), and (14) have only constant coefficients. Hence, the standard equation for hydrostatic inertia–gravity waves in a fluid at rest can be obtained (taking $\partial_z[D_\gamma(7) + f(11)]$) using Eqs. (1b), (1d), and (14).

If we consider the case where the waves have smaller spatial scales than the primary flow ($\gamma = \epsilon$), then the rhs of Eqs. (7), (11), and (14) contain coefficients that vary in time and space. However, the spatial and temporal derivatives of $\overline{\mathbf{u}}_H$ or $\overline{\theta}$ will be smaller than those of the secondary fields, so we will neglect the derivatives of $(\overline{\mathbf{u}}_H)$ or $\overline{\theta}$ and retain only the terms involving derivatives of the secondary fields.

Hence in both cases the wave equation can be written in dimensional variables:

$$[(D_\gamma^2 + f^2)\partial_{zz} + N^2\Delta_H]w^{*\prime} = -D_\gamma\partial_z\Delta_{\mathrm{NBE}} + f\partial_z\mathcal{A}_{1,\zeta^*} - \frac{g}{\theta_0}\Delta_H\mathcal{A}_{0,\theta^*}, \tag{15a}$$

$$\text{with}\quad \Delta_{\mathrm{NBE}} = \overline{\zeta}^* - \Delta_H\overline{\phi}^* - \beta\overline{u}^* - 2\mathcal{J}(\overline{v}^*, \overline{u}^*), \tag{15b}$$

$$\mathcal{A}_{1,\zeta^*} = (\partial_t + \overline{\mathbf{u}}_H{}^*\boldsymbol{\nabla})\overline{\zeta}^* + f\overline{\delta}^* + \beta\overline{v}^*, \quad \text{and} \tag{15c}$$

$$\mathcal{A}_{0,\theta^*} = (\partial_t + \overline{\mathbf{u}}^*\boldsymbol{\nabla})\overline{\theta}^* + \overline{w}^*\partial_z\Theta^*, \tag{15d}$$

with D_γ as defined in Eqs. (8) and $N^2 = g\theta_0^{-1}d\Theta^*/dz$.

The first term in the forcing consists in derivatives of the residual of the nonlinear balance equation. Previous numerical studies on the generation of inertia–gravity waves from baroclinic waves (Zhang 2004; Wang and Zhang 2007) have already shown qualitatively the relevance of this residual to spontaneously generated gravity waves: the residual has a maximum in the region where and at the time when gravity waves are generated. The above analysis provides a first quantitative justification for relating a diagnostic from the large-scale flow (involving the NBE residual) to the location and amplitudes of excited gravity waves.

It is not simply the residual of the nonlinear balance equation that appears as a forcing but the Lagrangian derivative of its vertical gradient. This will emphasize regions of strong vertical gradient of the residual and also regions where strong variations are seen along fluid trajectories. Our analysis further indicates additional terms that may play a role in the forcing of the gravity waves. These additional terms are residuals from the vorticity and from the potential temperature equation. The relative importance of the three terms, in practice, will need to be assessed in numerical simulations and case studies. Note, however, that in the case where the waves have smaller scales than the background flow, Eq. (15a) is more delicate to justify rigorously as the spatial derivatives of the terms on the left are greater than those on the right. Nevertheless, the above provides a guide for the forcing terms. To verify quantitatively the relevance of these forcing terms, numerical simulations of gravity wave generation in different baroclinic life cycles are underway (Wang and Zhang 2007; Plougonven and Snyder 2007). Further analysis of these different simulations will reveal whether the amplitudes of the excited gravity waves correlate in a systematic manner with the above forcing terms.

5. Conclusions and perspectives

The motivation of this note was to identify a diagnostic obtained from the large-scale flow that can be used quantitatively as an indicator of excitation of inertia–gravity waves (IGWs). The flow was described as a superposition of two types of motions: a primary flow that is essentially large-scale and balanced (nondivergent) and a secondary flow that is unbalanced and has smaller scales. Based on previous experience showing the key role that the residual of the nonlinear balance equation (NBE) could play (Zhang et al. 2001; Zhang 2004), it was chosen to use the divergence and the vorticity equations as the basic equation for the horizontal flow. Using scaling arguments, a wave equation, Eq. (15a), with forcing terms given by the primary flow was obtained in section 4.

Two cases were distinguished, depending on whether the waves had smaller spatial scales than the primary flow or not. If they did, the obtained equations were changed by replacing the time derivatives with Lagrangian derivatives taking into account the advection by the horizontal component of the primary flow.

Note that we have left open the definition of the separation of the flow. We have merely identified, using scaling arguments guided by observational and numerical experience, the form that the wave equation and its forcing terms should take. Work is underway to confirm the relevance of the above diagnostic by comparing systematically its amplitude and that of the excited gravity waves in different baroclinic life cycles.

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